The study of two-dimensional oscillations using a smartphone acceleration sensor: example of Lissajous curves

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Abstract
A smartphone acceleration sensor is used to study two-dimensional harmonic oscillations. The data recorded by the free android application, Accelerometer Toy, is used to determine the periods of oscillation by graphical analysis. Different patterns of the Lissajous curves resulting from the superposition of harmonic motions are illustrated for three experiments. This work introduces an example of how two-dimensional oscillations can be easily studied with a smartphone acceleration sensor.

1. Introduction

Many experiments using portable devices, for use in the teaching of physics, have been reported recently in the literature. Digital cameras [1], webcams [2], the optical mouse from computers [3, 4], wiimote [5] and other game console controllers [6] have been included. For instance, by using a simple digital camera [7, 8], a physics experiment can be carried out. The recorded videos allow us to measure time, distances and positions of objects.

The use of portable devices has recently been extended to smartphones. In particular, the acceleration sensor incorporated in smartphones has been used for the study of single and coupled oscillations at both qualitative and quantitative levels in high schools [9, 10] and in universities [11, 12], respectively. In this work, we extend the use of the smartphone acceleration sensor to the study of two-dimensional oscillations.

In fact, most oscillations in a student's everyday life and modern technology are more

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than one-dimensional. This is a major reason for including two-dimensional oscillation examples in physics teaching and taking advantage of the possibilities provided by the smartphone acceleration sensor which is a common device in daily use by students. As an amenable example, we have chosen to study Lissajous curves. Several works on Lissajous curves and physics teaching have been reported in the literature [13, 14]. For example, Lissajous curves are used to determine the frequency of a signal by combining them with another signal of known frequency.

Currently, the study of two-dimensional oscillations is carried out using somewhat tedious experiments. For example, in [15] the authors use an air table and a puck connected to it by springs. The trajectory of the puck is followed by the trace and described by it onto paper, which is later digitalized to extract the information of the trajectory, that is, $x(t)$ and $y(t)$. The introduction of the smartphone acceleration sensor in measuring two- and three-dimensional oscillations represents major progress in this respect since the instantaneous values of the acceleration are registered by the sensor along its three perpendicular axes. To the knowledge of the authors, this is the first work on two-dimensional oscillations using the smartphone acceleration sensor.

The outline of the paper is the following. In section 2 we describe the smartphone acceleration sensor and one of its free android applications, Accelerometer Toy. In section 3, we describe the experimental setup. In section 4, three different arrangements of two-dimensional oscillations are described. In the second and third of these experiments, the resulting mechanical Lissajous curves are shown. Finally, in section 5, some conclusions are drawn.

2. The smartphone acceleration sensor and android application

In our experiments we used the smartphone models Samsung Galaxy S2 with android 2.1 and LG-E510 with android version 2.3.4. The mass of the first smartphone is (0.1237 ± 0.0001) kg and of the second (0.1350 ± 0.0001) kg. The accelerometer sensor is based on three mutually perpendicular silicon circuits, each one oscillating in one direction, like a ball hanging on a spring whose movement is restricted to one direction. For the control of the accelerometer sensor the free android application 'Accelerometer Toy version 1.0.10' is used. This application takes 154 kB of memory and can be downloaded from the Google Play website [16]. The acceleration components a_x, a_y and a_z on the x, y and z axes, respectively, are registered by the sensor as a function of time. The precision in the measurement of the acceleration and time are $\delta a = 0.03$ m s^{-2}and $\delta t = 0.01$ s, respectively. This application also allows the user to save the output data to an ASCII file for further analysis. The structure of the ASCII file (figure 1) is the following. The first column is the order number of the time iteration, the second is the time in milliseconds, and the remaining three columns are the acceleration in the x, y and z axes (in m s^{-2}), respectively. Once the application is downloaded to the mobile device, a small test can be performed to ensure it is working correctly. It can be proved that when the mobile is left on a horizontal surface, the output curves for the acceleration exhibit values very close to zero for the x and y axes and ~9.8 m s^{-2} for the z axis.

3. Experimental setup

The experimental setup includes an air table, a tray to carry the smartphone and four springs. Three arrangements of the springs are used. The table was made of aluminium with an approximate total cost of ~40 euros. The socket for the air supply of this table allows coupling to conventional air suppliers included in the air tracks kits for basic physics laboratories. However, the fact that the experiments can be performed with the students' smartphones makes them more feasible and attractive.

The setup used for the experiments is shown in figure 2. The dimensions of the air table and the tray carrying the smartphone are also included. The table is coupled to an air supplier. When the air supplier is on, a thin layer of higher pressure air appears between the tray holding the smartphone and the surface of the table, allowing the tray to move with almost no friction in two dimensions. These conditions allow us to study two-dimensional harmonic oscillations by obtaining mechanical Lissajous curves.

Count	Time (ms)	X	Y	Z
1	0	0,21793	0,14982	9,83469
2	9	0,21793	0,12258	9,74494
3	19	0,17707	0,12258	9,86034
4	29	0,14982	0,12258	9,92445
5	39	0,1362	0,12258	9,86034
6	49	0,17707	0,10896	9,75776
7	59	0,1362	0,12258	9,73211
8	71	0,14982	0,08172	9,80905
9	82	0,14982	0,0681	9,74494
10	92	0,2043	0,04086	9,83469
11	105	0,23155	0,01362	9,83469
12	118	0,21793	0,01362	9,86034
13	126	0,24517	0,01362	9,82187
14	136	0,21793	-0,02724	9,86034
15	147	0,2043	-0,04086	9,80905
16	157	0,2043	-0,05448	9,80905
17	167	0,17707	-0,0681	9,73211
18	177	0,16344	-0,05448	9,69365
19	188	0,10896	-0,09534	9,80905
20	198	0,0681	-0,08172	9,77058
21	208	-0,02724	-0,08172	9,83469
22	218	-0,02724	-0,09534	9,83469
23	228	-0,05448	-0,08172	9,73211

Figure 1. Fragment of the output file of the Accelerometer Toy android application used for the experiments.

4. Two-dimensional experiments

4.1. Experiment 1

In the following experiment four springs of (10.5 ± 0.1) N m^{-1} are used, two for each of the x and y axes. The mass of the smartphone and the tray together is (0.1583 ± 0.0001) kg. After a diagonal shift along the $y = x$ curve, the smartphone starts oscillating approximately along this curve. In figure 3 (panel (a)) the oscillations of the acceleration along the x and y axes are shown. The resulting oscillation periods measured directly from the recorded data are shown in table 1. In panel (b), the curve resulting from the superposition of both harmonic motions is shown. It can be seen that it is almost a straight line, indicating the great similarity between the conditions of the oscillations of both axes ($T_x/T_y \simeq 1$).

4.2. Experiment 2

In the next experiment two springs of (46.9 ± 0.7) N m^{-1} are used along the x-axis and two springs of (10.5 ± 0.1) N m^{-1} along the y-axis. The mass of the smartphone and the tray together is (0.1583 ± 0.0001) kg. After a diagonal shift along the $y = x$ curve, the system starts oscillating. In figure 4 (panel (a)) the oscillations of the acceleration along the x and y axes are shown. The periods from the acceleration data are shown in table 1. In figure 4 (panel (b)), the curve resulting from the superposition of both harmonic motions is shown. The Lissajous curve corresponding to a ratio between the periods of $T_x/T_y = 0.6752 \simeq 2/3$ is shown.

4.3. Experiment 3

Finally, in this experiment two springs of (20.6 ± 0.1) N m^{-1} are used along the x-axis and two of (79.6 ± 0.7) N m^{-1} along the y-axis. The mass of the smartphone changes with respect to previous experiments and, along with the carrying tray, is (0.2145 ± 0.0001) kg. A diagonal shift along the $y = x$ curve is performed and the system starts oscillating. In figure 5 (panel (a)), the oscillations of the acceleration along the x- and y- axes are shown. The periods from the acceleration data are shown in table 1. In figure 5 (panel (b)), the Lissajous curve corresponding to a ratio between the periods of $T_x/T_y = 0.727 \simeq 8/11$ is shown.

The resulting periods in Experiments 2 and 3 differ slightly from the theoretical values expected for small oscillations of a body connected to

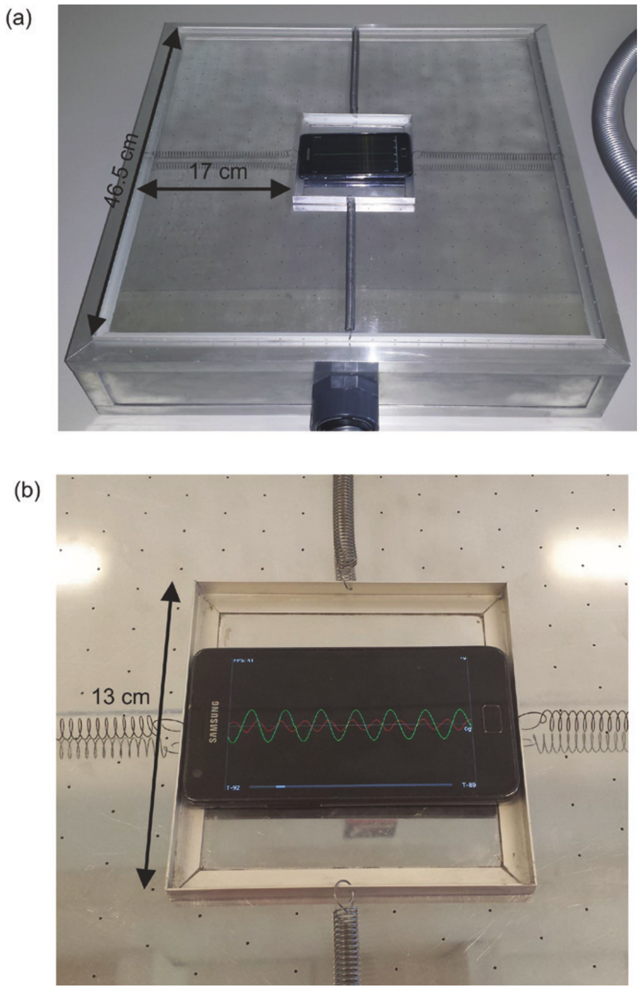

Figure 2. Experimental setup used for the experiments. In panel (a) a global view of the squared air table is shown. In panel (b), the detail of the smartphone and the squared tray holding it are shown. The dimensions of the air table and the tray are indicated on the picture.

two springs on a single axis, $T = 2\pi\sqrt{m/k}$. The springs, connected to the body along the perpendicular axis, have an influence on these oscillations since components of the elastic force appear on the oscillation direction. This is more stressed the greater the amplitude of the oscillation. This influences the results in that an effective force constant which is greater than the initial one appears. However, the resulting frequencies are larger than those expected from the theory. The experimental Lissajous curves resulting from Experiments 2 and 3 can be tested against the theoretical results (see figure 6) for $T_x/T_y \simeq 2/3$ and $T_x/T_y \simeq 8/11$, respectively (see the virtual laboratory implemented in [17]).

Table 1. Periods of oscillation from the graphical analysis of the recorded data.

		$(T \pm 0.01)$ s
Experiment 1	x	0.48
	y	0.48
Experiment 2	x	0.27
	y	0.40
Experiment 3	x	0.24
	y	0.33

5. Conclusions

Two-dimensional harmonic oscillations are studied using an air table and a smartphone acceleration sensor. The instantaneous acceleration data recorded by the sensor are plotted to obtain the

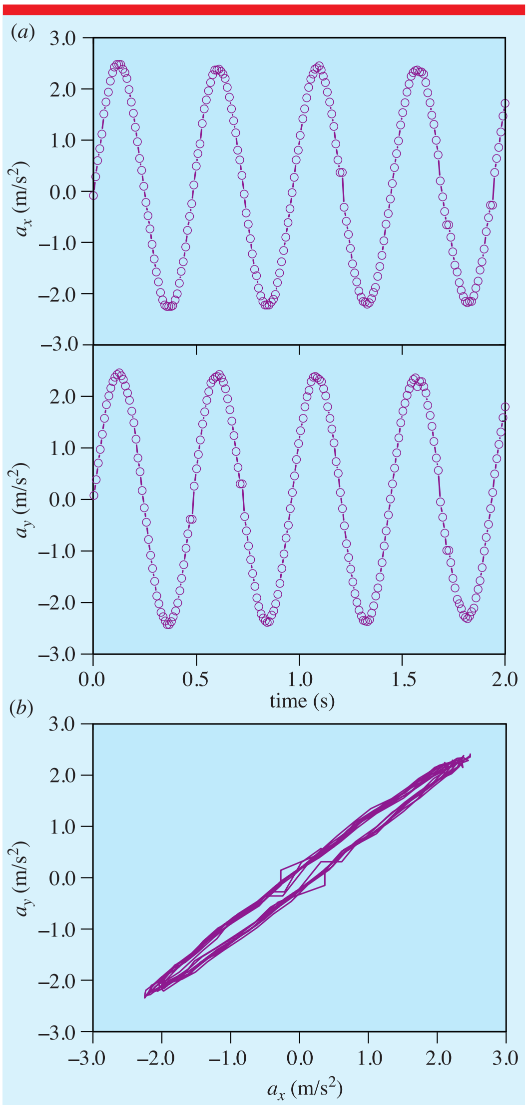

Figure 3. Results for Experiment 1 of (a) acceleration versus time on the x- and y-axes and (b) the corresponding Lissajous curve from $a_y = f(a_x)$.

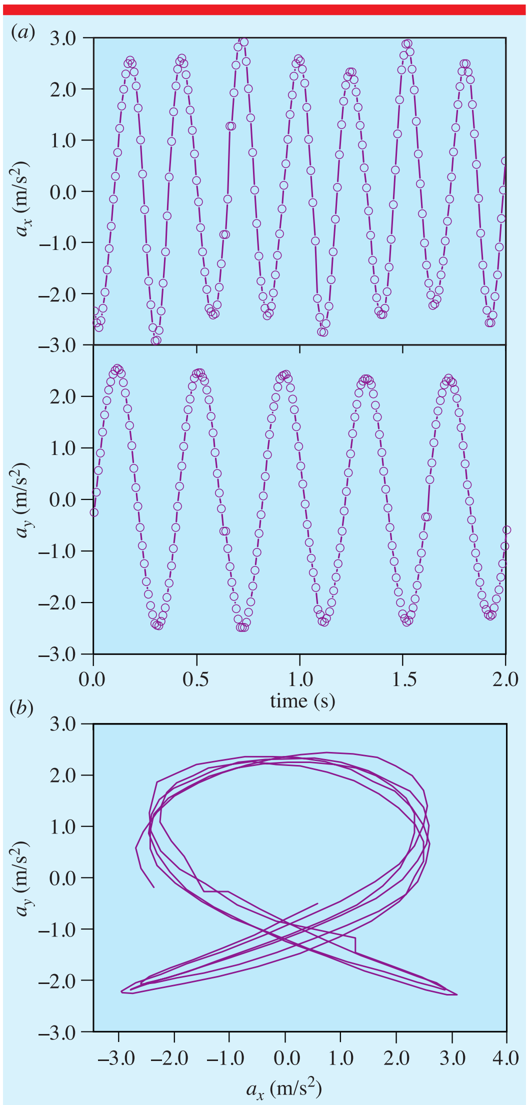

Figure 4. Results for Experiment 2 of the acceleration versus time on the x- and y-axes are shown in panel (a), and the Lissajous curve from $a_y = f(a_x)$ in panel (b).

periods of oscillation in the x- and y- axes. From the data of the harmonic oscillations, Lissajous curves are obtained for a ratio of the periods T_x/T_y of ~2/3 and ~8/11, for the second and third experiments, respectively. A very good agreement is obtained between the theoretical and experimental results. It is remarkable the progress that the use of the smartphone acceleration sensor represents in the study of two-dimensional oscillations; it is so much easier than previous methods. Moreover, it is an interesting way to extend the use of smartphones, a very familiar device for the students, beyond the standard use for communication. We think that whenever physics concepts are linked to popular aspects of students' everyday lives, there is a clear and positive impact on motivation.

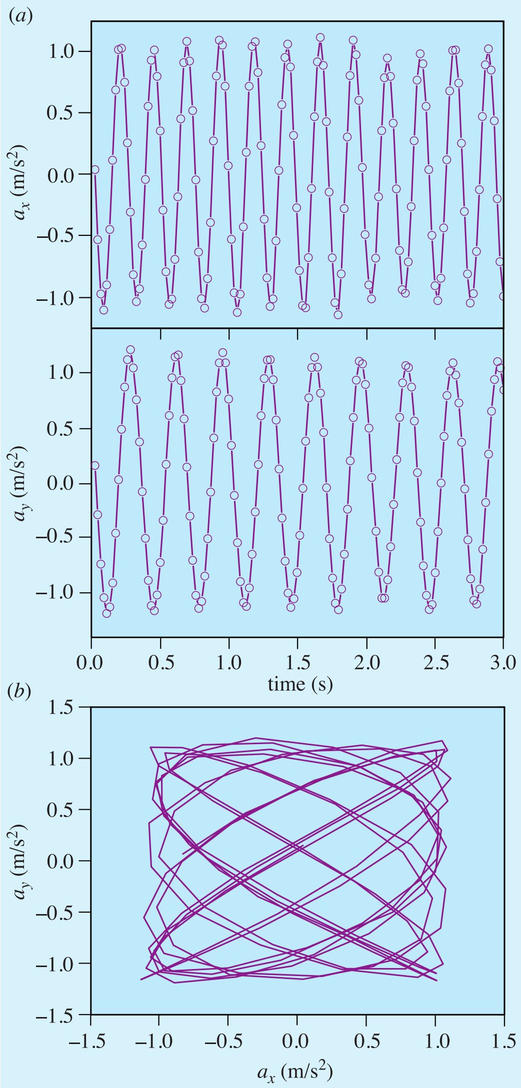

Figure 5. Results for Experiment 3 of the acceleration versus time on the x- and y- axes are shown in panel (a), and the Lissajous curve from $a_y = f(a_x)$ in panel (b).

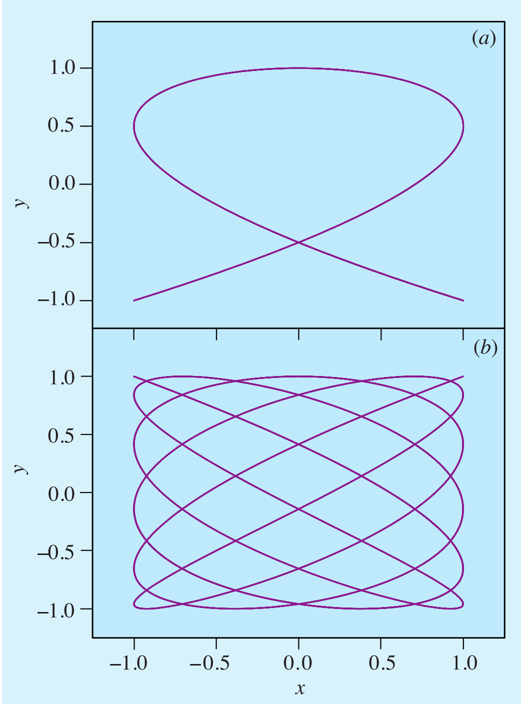

Figure 6. Theoretical Lissajous curves for $T_x/T_y = 2/3$ in panel (a) and $T_x/T_y = 8/11$ in panel (b). In both cases an amplitude of 1 arbitrary unit has been considered. The phase difference between x and y oscillations is 0 degrees.

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