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A quantitative analysis of coupled oscillations using mobile accelerometer sensors

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Abstract
In this paper, smartphone acceleration sensors were used to perform a quantitative analysis of mechanical coupled oscillations. Symmetric and asymmetric normal modes were studied separately in the first two experiments. In the third, a coupled oscillation was studied as a combination of the normal modes. Results indicate that acceleration sensors of smartphones, which are very familiar to students, represent valuable measurement instruments for introductory and first-year physics courses.

(Some figures may appear in colour only in the online journal)

1. Introduction

Coupled oscillators can be found in a wide range of phenomena in nature and technology, such as vibrations of atoms in solids [1]. These systems deserve attention on physics and engineering courses, which is why they can be found in most general physics textbooks [2]. The formalism of coupled oscillations has also been used to introduce more complex concepts, for instance in quantum mechanics [3]. Additionally, these systems are also important from a mathematical perspective because they represent coupled differential equations whose solution involves calculus and linear algebra concepts.

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Many efforts have been made to introduce coupled oscillations as classroom experiments, usually based on bodies attached to pendulums and springs [4–8]. The position of the oscillating body is usually followed using image recognition techniques (IRT) [6, 8]. In rotatory systems, the oscillations are usually followed by measuring the angular velocity [9]. Classroom experiments on oscillations can also be performed in two dimensions using an air table [10]. Magnetic fields as coupling elements are also found in systems of coupled oscillators [11–13]. Even more exotic examples have been introduced in physics teaching, for example studying the sound generated by vibrating wine glasses in [14].

Recently, in [15], the authors applied digital video images to the study of damped coupled oscillations. Two carts were mounted on an air track and connected by springs. For the localization of the carts at each moment of time, IRTs were used. Computer software based on Fourier transforms was developed to perform the calculations. The results indicate that IRTs are a powerful tool to study this kind of system. However, the use of IRTs, even though they are precise, may produce results that are tedious and difficult to understand for students. It uses advanced computer techniques and mathematical concepts that go beyond those appropriate for introductory and first-year university courses.

In this paper we present a simpler alternative for measuring the instantaneous acceleration of the carts at each moment of time, the use of smartphone acceleration sensors. The use of this type of sensor in the study of simple and coupled pendulum oscillations has been introduced qualitatively at high-school level [16, 17]. In this work, we performed a detailed quantitative analysis of a system formed by two coupled oscillators. The acceleration sensors of two smartphones were used. In order to record the data captured by the sensors, the 'Accelerometer Monitor version 1.5.0' Android application[7] was used. Smartphones are very familiar to students and our proposal finds suitable applications for them in introductory, and first-year university physics courses.

The outline of the paper as follows. In section 2, the experimental set-up is described. In section 3, a theoretical overview of the coupled oscillations is given. In section 4 three cases are studied; symmetric oscillations (section 4.1), asymmetric normal mode oscillations (section 4.2) and non-normal oscillations (section 4.3) as a combination of normal modes. Finally, in section 5 conclusions are drawn.

2. Experimental set-up

The experimental set-up is shown in figure 1. Two gliders of equal mass, $m = (500.5 \pm 0.1)$ g, are placed on the air track. This value includes the mass of the glider itself, the smartphone and compensating weights for each case. Three springs of force constant $k_0 = (47.1 \pm 0.2)$ N m^{-1} are used. This value was obtained by measuring the shift (x) produced after hanging a mass from the spring since $mg = kx$. Six values of mass between 200 and 600 g and a least square algorithm were used to determine the aforementioned value of the force constant as the slope of the fitting linear function. Two springs connect each glider to the fixed ends at both sides and another connects both gliders directly. When the air supply is on the air flows through small holes on the track surface, producing a layer of air between the track and the gliders, which allows gliders to move with almost no friction.

Mobile sensors are used to measure the oscillations. For this purpose, the mobile devices are mounted each on each glider. The mobile mounted on the left glider is a smartphone (LG-E510 with Android version 2.3.4) and the one mounted on the right is also a smartphone (Samsung Galaxy S1GT-i9000 with Android version 2.3.5). For the interaction with the mobile

[7] https://play.google.com/store/apps

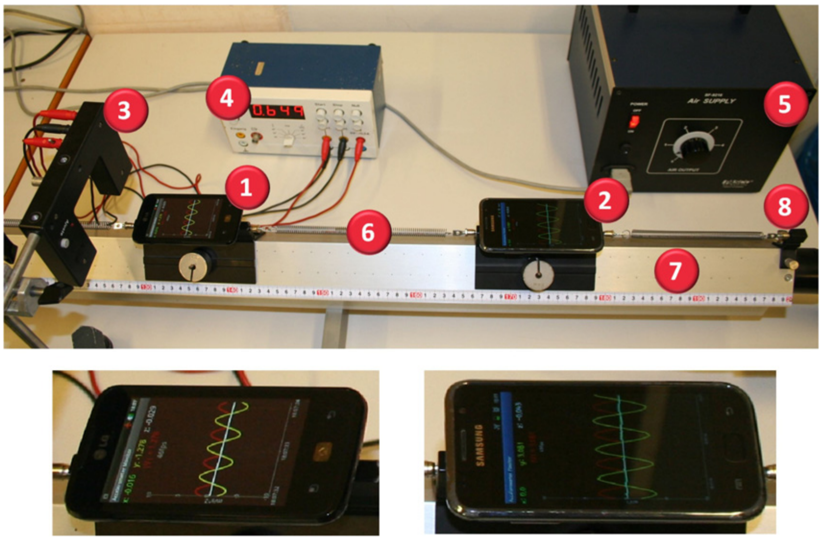

Figure 1. Experimental set-up used in the experiments. In the upper part of the figure (1) and (2) are the smartphones, (3) is the photometer, (4) is the digital counter of the photometer, (5) is the air supply, (6) is the central spring and (7) is the right-hand fixed end. In the lower part of the figure, the smartphone views are rendered in close-up. The yellow line represents the instantaneous acceleration of the mobile.

sensor, the free Android application 'Accelerometer Monitor version 1.5.0' was used. This application takes 348 kB of SD card memory and can be downloaded from the Google play website (see footnote 7). This application reports the vibrations of the mobile in real time by registering the acceleration components on x, y and z axes at each time step. The resolution of the mobile phone in the measurement of the acceleration is $\delta a = 0.011\,97$ m s^{-2} and the average sampling time is $\delta t = 0.02$ s. Since the oscillations take place along the y-axis, the values of the acceleration for the x and z-axis remain very close to zero during the measurement process. This application also allows an output file to be saved, from which the data can be retrieved for further analysis. Once the application has been downloaded to the mobile device, a small test can be done to test the functioning of the sensor. If the mobile is left in rest on a horizontal surface, the application output curves for the acceleration should indicate values very close to zero for all axes.

3. Theoretical overview

In general, the equation of motion of the oscillating gliders can be written as follows:

$$m x_1 = -k_0 x_1 - k_0 (x_1 - x_2) \tag{1}$$

$$m x_2 = -k_0 x_2 - k_0 (x_2 - x_1), \tag{2}$$

where, m is the mass of each body (glider + mobile + compensation weights), x_1 and x_2 are the displacements at each moment of time (positive to the right) with respect to the equilibrium position and k_0 is the spring constant of the springs.

The system of equations above can be uncoupled by introducing a new set of coordinates:

$$q_1 = (x_1 + x_2) \quad \text{and} \quad q_2 = (x_2 - x_1), \tag{3}$$

Table 1. Parameters (and their standard uncertainties in parentheses) of the fit of symmetric mode oscillation accelerations to equations (9) and (10) with $A_2 = 0$. The square of the curvilinear correlation coefficient, R^2 for both fits has also been included.

	Glider 1	Glider 2
A_1 (m)	0.065 60(18)	0.066 20(24)
ω_1 (rad s^{-1})	9.6495(15)	9.6511(20)
ϕ_1 (rad)	1.5236(52)	1.4810(72)
R^2	0.998	0.996

resulting in the following form for equations (1) and (2),

$$mq_1 + k_0 q_1 = 0, \tag{4}$$

$$mq_2 + 3k_0 q_2 = 0. \tag{5}$$

Equations (4) and (5) represent the motion of two harmonic oscillators. The solutions are written as:

$$q_1 = A_1 \sin(\omega_1 t + \phi_1), \tag{6}$$

$$q_2 = A_2 \sin(\omega_2 t + \phi_2), \tag{7}$$

where $\omega_1 = \sqrt{k_0/m}$ and $\omega_2 = \sqrt{3k_0/m}$ are the angular frequencies for each normal mode and ϕ_1 and ϕ_2, their corresponding initial phases.

If we invert equations (3), we get,

$$x_1 = \tfrac{1}{2}(q_1 + q_2) \quad \text{and} \quad x_2 = \tfrac{1}{2}(q_1 - q_2). \tag{8}$$

The functional form of the accelerations, a_1 and a_2, of the mobile phones can be obtained by substituting equations (6), (7) in (8) and by taking the second derivatives of x_1 and x_2, as follows:

$$a_1 = \frac{d^2 x_1}{dt^2} = -\frac{1}{2}\left[A_1\omega_1^2 \sin(\omega_1 t + \phi_1) + A_2\omega_2^2 \sin(\omega_2 t + \phi_2)\right], \tag{9}$$

$$a_2 = \frac{d^2 x_2}{dt^2} = -\frac{1}{2}\left[A_1\omega_1^2 \sin(\omega_1 t + \phi_1) - A_2\omega_2^2 \sin(\omega_2 t + \phi_2)\right]. \tag{10}$$

For the case of the symmetric and asymmetric modes, $A_2 = 0$ and $A_1 = 0$, respectively, in the above equations.

4. Results and discussion

4.1. Symmetric normal mode

The first experiment consisted of the measurement of the coupled system moving under symmetric oscillations (figure 2). In this respect, both gliders received initial displacements of $x_{01} = 6.5$ cm and $x_{02} = 6.5$ cm, respectively. The values of acceleration obtained from the mobile application are fitted to the equations (9) and (10) by making $A_2 = 0$. The parameters and their standard uncertainties were obtained using the nonlinear Levenberg–Marquardt [18, 19] fitting algorithm, implemented in ORIGIN ver 6.1[8] data analysis software. The resulting values are given in table 1. It can be seen that the square of the curvilinear correlation coefficient (R^2) in the last row is very good and indicates the quality of the fitting.

[8] http://www.originlab.com/

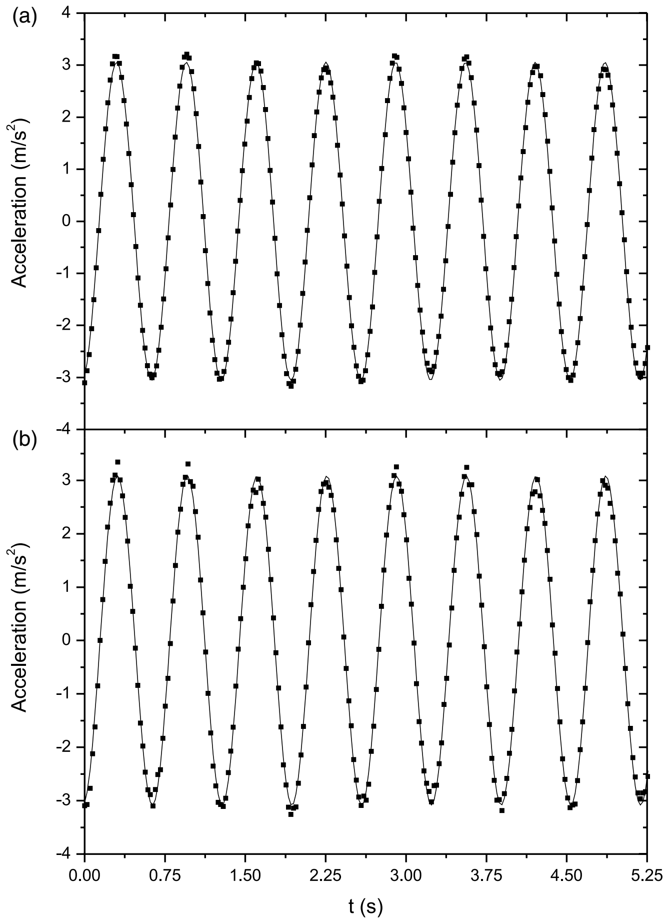

Figure 2. Accelerations of gliders 1 (a) and 2 (b) for the symmetric mode oscillations. The data points are indicated by a dotted line and the fit by a solid line.

The period of oscillation of the gliders can be obtained from the average value of the fitted frequencies ($\widetilde{\omega}_1$) (table 1) as $T_{\rm fit} = 2\pi/\widetilde{\omega}_1$ yielding (0.6511 ± 0.0001) s. This value has been compared to the value obtained with the photometer (figure 1, component 3), (0.650 ± 0.001) s, yielding a very low discrepancy of 0.17%. On the other hand, the spring constant has been obtained as $k = m\widetilde{\omega}_1^2 = (46.61 \pm 0.02)$ N m^{-1} and compared to $k_0 = (47.1 \pm 0.2)$ N m^{-1} (see section 2), yielding a discrepancy of 1.0%.

4.2. Asymmetric normal mode

For the antisymmetric normal mode (figure 3), the initial displacements of the gliders were $x_{01} = -2$ cm and $x_{02} = 2$ cm, respectively. In table 2, the parameters of the fit to equations (9) and (10) were registered. The value for the amplitude, $A_1 = 0$, has been used for this case.

In the same way as for the symmetric case, the oscillation periods for the gliders have been obtained, resulting in $T_{\rm fit} = 2\pi/\widetilde{\omega}_2 = (0.3750 \pm 0.0002)$ s. The value obtained with the photometer is (0.375 ± 0.001) s. These values match within the uncertainty interval.

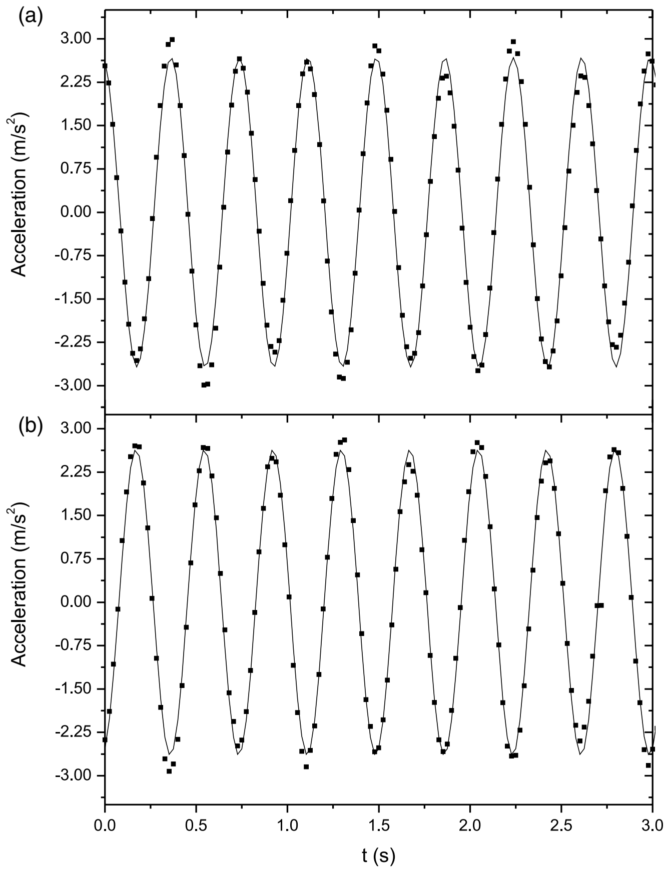

Figure 3. Acceleration of gliders 1 (a) and 2 (b) for the asymmetric mode oscillations. The data points are indicated by a dotted line and the fit by a solid line.

Table 2. Parameters (and their standard uncertainties in parentheses) of the fit of asymmetric mode oscillation accelerations to equations (9), and (10) with $A_1 = 0$.

	Glider 1	Glider 2
A_2 (m)	0.019 10(21)	0.018 80(23)
ω_2 (rad s^{-1})	16.7553(81)	16.7570(75)
ϕ_2 (rad)	1.529(17)	1.473(15)
R^2	0.988	0.990

The calculated value of the spring constant is $k = (46.84 \pm 0.13)$ N m^{-1} and the resulting discrepancy with k_0 is 0.55%.

4.3. Combination of symmetric and asymmetric modes

In the third experiment, coupled oscillations are studied as a combination of symmetric and asymmetric modes (figure 4). For this purpose, the glider on the right-hand side was kept at rest ($x_{02} = 0$ cm), while the glider on the left-hand side was displaced in $x_{01} = -3$ cm. In

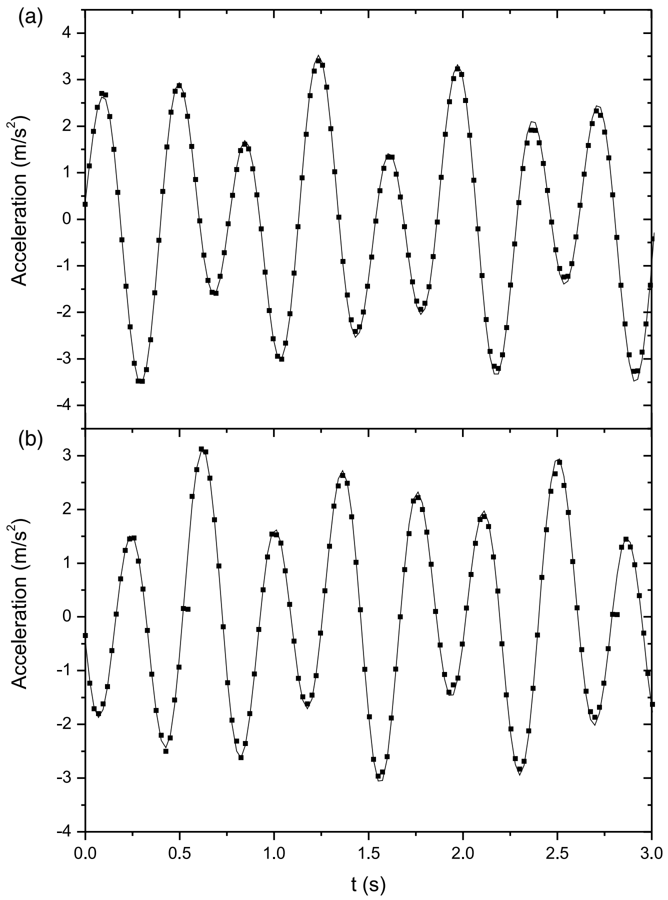

Figure 4. Acceleration of gliders 1 (a) and 2 (b) for an oscillation resulting from the combination of symmetric and asymmetric modes. The data points are indicated by a dotted line and the fit by a solid line.

Table 3. Parameters (and their standard uncertainties in parentheses) of the fit of the acceleration of coupled oscillations to equations (9) and (10) are shown.

	Glider 1	Glider 2
A_1 (m)	0.023 20(23)	0.017 70(21)
A_2 (m)	0.017 60(11)	0.016 30(71)
ω_1 (rad s^{-1})	9.6490(52)	9.6571(57)
ω_2 (rad s^{-1})	16.7719(21)	16.7706(21)
ϕ_1 (rad)	−1.857(19)	−1.513(23)
ϕ_2 (rad)	0.1190(83)	0.1470(82)
R^2	0.995 26	0.995 35

table 3, the parameters of the fit and their standard uncertainties are shown. It can be seen that the frequencies in table 3 for each glider are in good agreement with those for the symmetric and asymmetric modes in subsections 4.1 and 4.2. In both cases the value of R^2 is greater than 0.995, which indicates the good quality of the fitting procedure.

5. Conclusion

Coupled oscillations have been studied using smartphone acceleration sensors. Symmetric and asymmetric normal modes have been analyzed separately. A general coupled oscillation has been studied as a combination of the normal modes. The discrepancies between fitted and experimental values for the period and the spring constant are less than 1% for all cases. The results indicate that the mobile sensor is capable of being used as a reliable instrument to measure rapid variations of the instantaneous acceleration components in physics phenomena such as oscillations. This experiment is being implemented as a laboratory experiment in the first physics course for the Industrial Design Engineering degree at the Universitat Politècnica de València, Spain. A preliminary survey indicated that 96% of the students carry a smartphone. The acceleration sensor of this type of device, which is very familiar to students, finds very suitable applications in introductory and first-year university physics courses.

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