A Virtual Laboratory Designed For Teaching Diffractive Lenses

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Abstract

En este trabajo presentamos un laboratorio virtual desarrollado en Matlab Gui© *(Interfaz Gráfica con el Usuario) para su uso en los cursos de Óptica como herramienta informática para la enseñanza de las propiedades de focalización de las lentes difractivas. Esta interfaz GUI permite a los estudiantes aprender fácil y rápidamente la influencia de los parámetros constructivos de la lente en el volumen focal. A modo de ejemplo analizamos en este trabajo lentes difractivas fractales porque hemos encontrado que la geometría fractal es un tema muy motivador para los estudiantes ya que está relacionado con un gran número de fenómenos cinetíficos y tecnológicos.*

We present a virtual laboratory generated in Matlab Gui© *(Graphical User Interface) for its use in Optics courses as an informatic tool for teaching the focusing properties of a diffractive lens. This* Gui *allows the students to learn easily and rapidly about the influence on the focal volume of the lens construction parameters. As an example in this work we analyze fractal diffractive lenses because we found that fractal geometry is a highly motivating topic for students since it is related to a wide range of scientific and technological phenomena.*

Keywords: Educational Software, Matlab©, Fractals, Optics.

1   Introduction

At present, Information and Communication Technologies (ICTs) are widely used in teaching science. Traditional content-oriented teaching approaches are being replaced by student-oriented approaches. In this context the teacher acts as a guide, giving the students the tools and showing them the different ways they can apply themselves to the learning process. The teacher has to choose the best way to allow the student to construct and understand the solution to practical problems. This tendency is named constructivism [1]. The teacher does not transfer the knowledge to the students, but helps them to construct, or at least interpret, the reality of the body of knowledge based on their (real or virtual) experiences. There is a wide consensus in the natural science teacher's community that this way of learning is much better for students both in terms of understanding and visualizing rather than just attending to lectures, because it permits a closer interaction between the students and the matter of study. Within this context, learning processes based on digital simulations have demonstrated to provide several advantages [2], although its pedagogical effectiveness is conditioned to a complete integration of the simulation in the global instruction plan [3]. The introduction of ICTs for teaching has the advantage of offering the possibility of showing the matter of study from different points of view. Concretely, computer simulations allow to present virtual-interactive experiments to the students where they can decide which are the correct values of certain parameters to obtain and visualize a given physical phenomena [4, 5]. Within this context, in this paper we present a new virtual laboratory generated in MATLAB GUI© (Graphical User Interface) for its application in Optics courses as an educational tool for visualizing properties of diffractive binary lenses [6]. The software is suitable to be applied in both in Engineering and Physics degrees, as for example in the course "Diffraction in optical devices" of the physics degree at the University of Valencia (Spain), where it is currently under test. This kind of lenses are key elements for obtaining images in several scientific and technological areas such as, THz tomography, astronomy and soft X-ray microscopy [7]. The GUI we have developed is characterized by its friendly appearance and allows the students to learn easily and rapidly about the focusing properties of diffractive lenses with different geometries. From a didactic point of view, the developed graphical interface is very stimulating because it also becomes a way for introducing students into basic research activities. In fact, to show the versatility of our proposal, in this paper we present the analysis of the recently proposed fractal zone plates (Fzps) [8].

2   Theory

Let us start by considering the irradiance at a given point on the optical axis, provided by a rotationally invariant pupil function described by $p(r)$, illuminated by a monochromatic plane wave. Within the Fresnel approximation, this magnitude is given as a function of the axial distance from the pupil plane z, as [9]

$$I(z) = \left(\frac{2\pi}{\lambda z}\right)^2 \left| \int_0^a p(r_0) \exp(-i\frac{\pi}{\lambda z}r_0^2)r_0 dr_0 \right| \qquad (3.1)$$

For our purposes it is convenient to express the pupil transmittance as function of a new variable defined as

$$\varsigma = \frac{r_0}{a} - 0.5 \qquad (3.2)$$

in such a way that $q(\varsigma) = p(r_0)$. By using the dimensionless axial coordinate $u = \frac{a^2}{2\lambda z}$, the irradiance along the optical axis can be now expressed as

$$I(u) = 4\pi^2 u^2 \left| \int_{-0.5}^{0.5} q(\varsigma) \exp(-\imath 2\pi u\varsigma) d\varsigma \right| \tag{3.3}$$

In Eq. 3.1, a is the maximum extent of the pupil function, and λ is the wavelength of the light. From this result it is straightforward to note that the behavior of $I(u)$ is basically governed by the square modulus of the Fourier transform of $q(\varsigma)$. In recent years, the study of fractals has attracted the attention of researchers, encouraged by the fact that many physical phenomena, natural structures and statistical processes can be analyzed and described by using a fractal approach [10] . Let us now consider a pupil function $q(\varsigma)$ that holds a fractal structure. Thus, from well-known properties of fractals and their Fourier transforms, it is direct to conclude that such element will provide an irradiance along the optical axis with a fractal profile. We called this kind of pupils fractal zone plates (FZPs) because, as we will see next, in some cases they can be constructed from conventional Fresnel zone plates. Thus a FZP is a zone plate characterized by its fractal profile along the square of the radial coordinate. When it is illuminated by a parallel wavefront aFZP it produces multiple foci along the optical axis. The internal structure of each focus exhibits a characteristic fractal structure, reproducing the self-similarity of the originating FZP [8]. Although in the construction of a FZP any fractal 1-D structure can be used, we will focus our attention on binary Cantor sets.
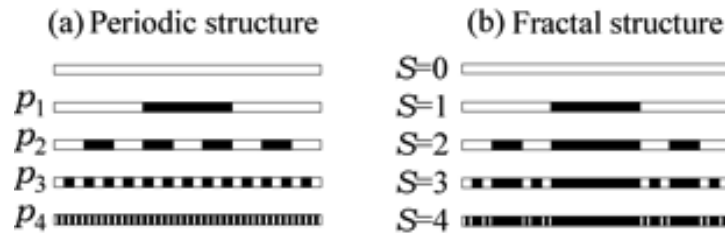

Figure 3.1: Schemes for the generation of the binary function $q(\varsigma)$ for: (a) Fresnel zone plate with different periods $p_S = p(N, S)$, for $N = 2$ and different values of S; (b) its associated FZP. In this representation clear and dark segments correspond to the values 1 and 0 of the generating binary function, respectively.

As it is well known, a binary Fresnel zone plate [7] consists of alternately transparent and opaque zones whose radii are proportional to the square root of the natural numbers. By using Eq.3.2, it is easy to obtain that the function $q(\varsigma)$ for these pupils is a Ronchi-type periodic binary function with period p (see Figure 6.1a), that can be written as

$$q(\varsigma) = q_{ZP}(\varsigma, p) = rect(\varsigma)rect\left(\frac{mod\left(\varsigma + \frac{p-1}{2}, p\right)}{p} \right), \tag{3.4}$$

where the function $mod(x, y)$ gives the remainder on division of x by y and $rect(x)$ is the rectangle function that is equal to unity for x less than half, and zero otherwise.In a similar way, FZP s are constructed by replacing the above 1-D periodic function by a 1-D binary function with fractal profile. Consider, for example, the particular case of a regular Cantor fractal whose construction procedure is shown in Figure 6.1b). In the first stage ($S = 1$), the initial segment is divided into an odd number of segments $2N - 1$ and the segments in the even positions are removed (in the figure a triadic Cantor set was considered, thus $2N - 1 = 3$). For

the remaining N segments at the first stage, this " slicing and removing" process is repeated in the second stage and so on. In mathematical terms, the FZP transmittance function, developed up to a certain "growing" stage S, can be expressed as the product of the periodic functions $q(\varsigma)$ in Eq. (4) as

$$q(\varsigma) = q_{FZP}(\varsigma, N, S) = \prod_{i=0}^{s} q_{ZP}\left(\varsigma, \frac{2}{(2N-1)^i}\right) \tag{3.5}$$

It is instructive to note that the FZP in Eq.3.5 can be understood as an associated Fresnel zone plate $q_{ZP}(\varsigma p(N, S))$, with period

$$p(N, S) = \frac{2}{(2N-1)^s} \tag{3.6}$$

but with some missing clear zones (compare Figure 6.1a with Figure 6.1 1b). Figure 6.2 shows a FZP generated from a triadic Cantor set, up to S=3, and the corresponding Fresnel zone plate with period $p(2, 3)$.
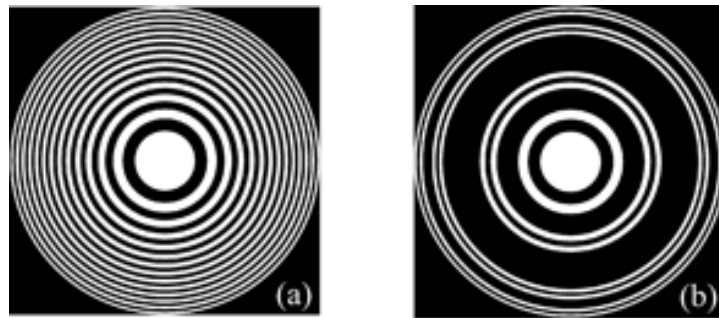

Figure 3.2: Fresnel zone plate (a) and the associated FZP (b) generated from the 1-D functions in Figure 6.1 for $S = 3$. The generating process consists in rotating the respective whole structure around one extreme after the change of variables in Eq. 3.2.

3   Graphical interface

Figure 6.3 shows the graphical user interface GUI implemented in MATLAB$^{©}$ for the computation of the axial irradiance generated by binary diffractive lenses (Eq. 3.3). This MATLAB$^{©}$ software presents a visual programming environment to perform and run programs that requires a continued flow of data. It presents the basic features of programs like VISUAL BASIC$^{©}$ or VISUAL C++$^{©}$, but much more affordable for teaching. The application we propose consists of two files, namely: **ZP design.m** and **ZP design.fig**. The first is the executable and the second is the graphic part. These files have been developed with MATLAB$^{©}$ version 7.9.0529 (R2009b) and an online version of the software for it use for non-presential teaching is available in [6].

To start the simulation program, the transmittance of the binary diffractive lens is entered in the upper window in binary mode or decimal. In binary mode, the introduced set represents the sequence of transparent and opaque of ring areas of the lens. For example, the sequence of Cantor for $N = 2, S = 3$, (see Figure 6.1b) is codified as 101000101 and the equivalent periodic
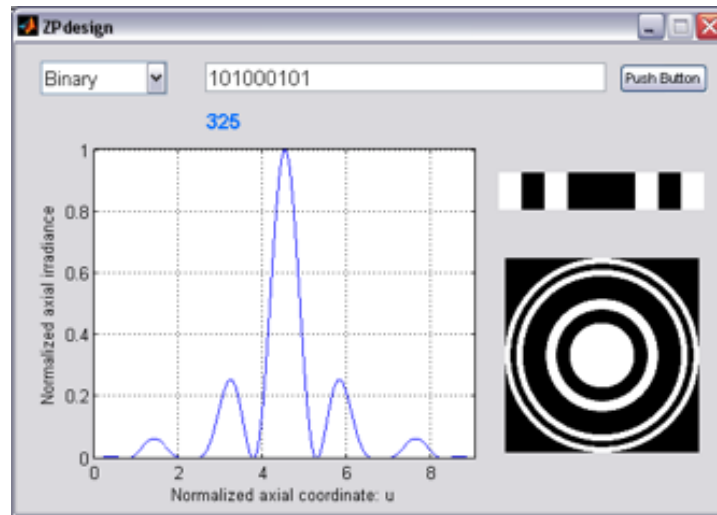
Figure 3.3: Graphical Interface Gui implemented for the computation of the axial irradiance diffractive lenses.

structure as 101010101 (see Figure 6.1a). In decimal format, the program first translates the entered number to binary code. So, decimal number 325 would represent the sequence above fractal, while the 341 would represent the equivalent periodic sequence. In the upper right window, the program shows the codified function $q(\varsigma)$ (white for $q(\varsigma) = 1$ and black for $q(\varsigma) = 0$). With the change of variables of Eq. 3.2, the program shows in the lower right part of the frame the transmitance of the considered binary lens. The function $q(\varsigma)$ it is constructed from the values $\varsigma_j := -0.5 + \frac{j}{n}$ for $j = 0, 1, \cdots, n$ where n it is the number of zones (transparent and opaque) of the lens, so that ,

$$q = \sum_{j=1}^{n} q_j \chi_{[\varsigma_{j-1}, \varsigma_j]}, \tag{3.7}$$

where,

$$\chi_{[\varsigma_{j-1}, \varsigma_j]} = \begin{cases} 0, & \text{if } \varsigma \notin [\varsigma_{j-1}, \varsigma_j]; \\ 1, & \text{if } \varsigma \in [\varsigma_{j-1}, \varsigma_j], \end{cases} \tag{3.8}$$

and q_j it takes values 0 for opaque zones or 1 for the transparent zones. In the program Eq. 3.7 has been used for the transmittance function in the computation of the axial irradiance $I(u)$ Eq. 3.3, i.e.:

$$\int_{-0.5}^{0.5} q(\varsigma) \exp(-\imath 2\pi u \varsigma) d\varsigma = \sum_{i=1}^{n} \frac{\imath p_j}{2\pi u} \left(\exp(-\imath 2\pi u \varsigma_j) - \exp(-\imath 2\pi u \varsigma_{j-1}) \right) \tag{3.9}$$

The axial irradiance, $I(u)$, shows the location of the different foci of the zonal plate. The axial extension of each focus is a periodic function in the normalized variable u whose period will coincide with the total number of areas introduced. However, for simplicity, the program only shows the normalized axial irradiance for the first period of the function corresponding to the main focus of the diffractive lens which satisfies the following relationship

$$f(N, S) = \frac{a^2}{\lambda(2N - 1)^S},\tag{3.10}$$

both for a FZP as for its equivalent Fresnel ZP.

sectionExamples The axial irradiance of the Fzp computed for different stages of growth S and for $N = 2$ is shown in Figure 6.4. The irradiance of the associated Fresnel zone plate is shown in the same figure for comparison. As in the case of a Fresnel zone plate, the axial irradiance behavior of the Fzp can easily be interpreted as the interference between the successive rings over the pupil. Note that the scale for the axial coordinate in each step is a magnified version of the one in the previous step by a factor $2N - 1 = 3$. It can be seen that the axial positions of the central lobes of the foci coincide with those of the associated Fresnel zone. It is clear that, while the internal structure of each focus in the Fresnel zone plate vanish progressively, the axial response for the Fzp exhibits its characteristic fractal profile In fact, the patterns in the upper part of Figure 6.4 are self-similar (the axial irradiance for a given stage S is a modulated version of that associated with the previous stage). We have called this scaling property along the optical axis, which holds for any N, the axial scale property. This means that the axial irradiance reproduce the self-similarity of the Fzp .

Figure 6.5 shows the axial response of the fractal lens with different number of segments N for a given order S. By increasing N, i.e.: considering the different sets of Cantor known as poliadic Cantor sets, it is obtained that the relative width of the main focus and the two secondary focus arising from the fractal structure on each side of it are reduced, this property can be interpreted in terms of the axial resolution the lens.

4   Conclusions

A Gui designed for teaching certain aspects of diffractive optics was presented. The examples clearly show how the developed Gui allows a visual analysis of the influence on the axial irradiance of the different design parameters of a binary diffractive lens. From a didactic point of view, the program allows exemplifying certain aspects of the theory of the diffraction, for example by using the interface it is possible to explain the appearance of the different maxima and minima on the axial irradiance as the result of the multiple interferences between the waves that they come of the different zones from the diffractive lens. In order to further motivate the students, non conventional diffractive lenses were analyzed, instead fractal configurations have been selected. In our own experience using the Gui software, we have observed interestingly that different aspects of the fractal foci produced by different configurations of the diffractive lens, which were assigned to different groups of students, produce stimulating discussions among them and, as a consequence, the students themselves request for more advanced tasks on fractals.

5   User's Guide for programs

- **zpdesigner.m**: This program can be run directly using the sentence "zpdesigner" in a Matlab© command-line: This command opens the input-output window. Selecting previously binary or decimal mode with the pop menu, the transmittance of the binary diffractive lens is entered in the upper window. Finally click the push bottom. The program
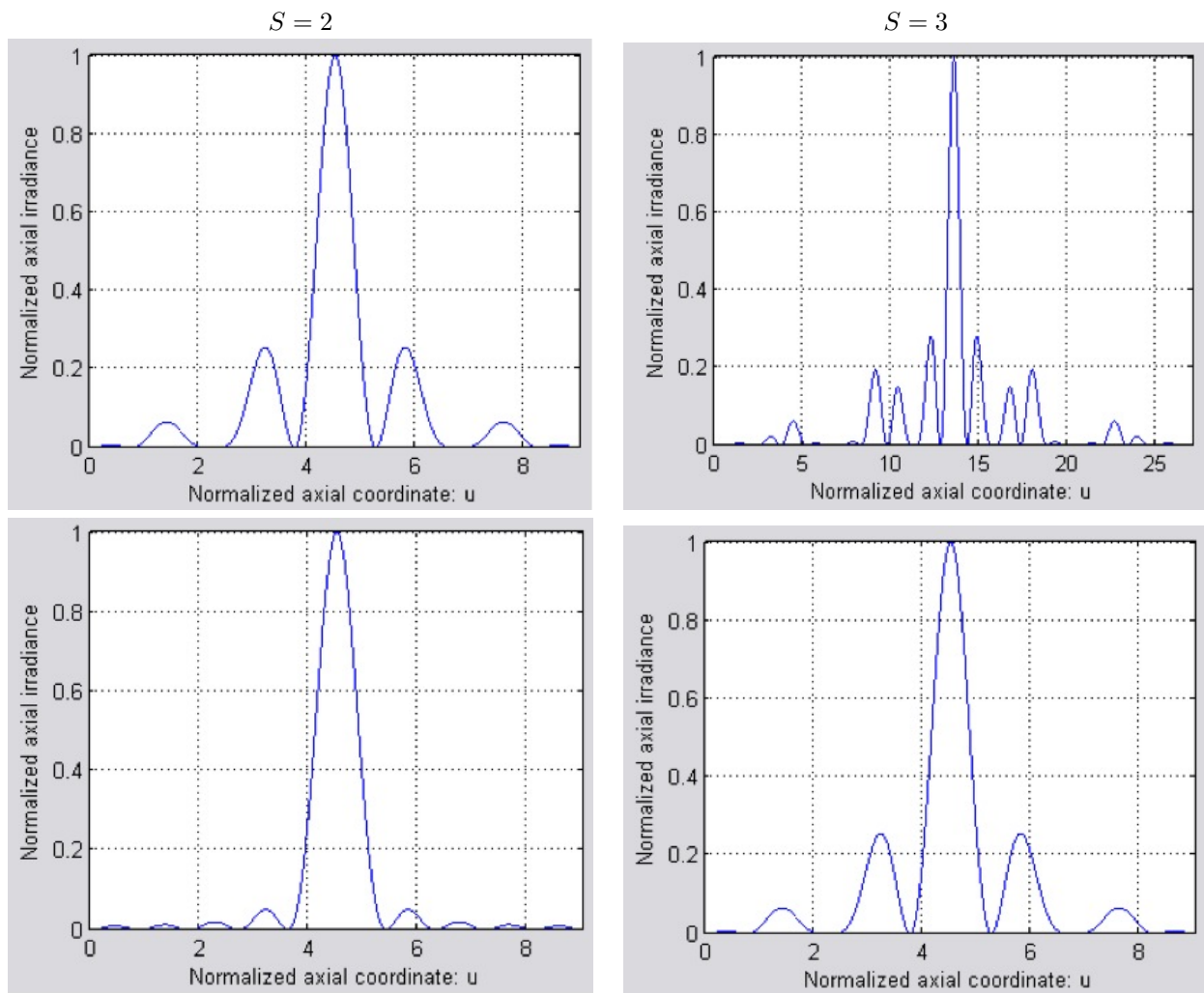
Figure 3.4: Normalized irradiance vs. the axial coordinate u obtained for a Fzp at two stages of growth (upper part) and for its associated Fresnel zone plate (lower part). In all cases $N = 2$.
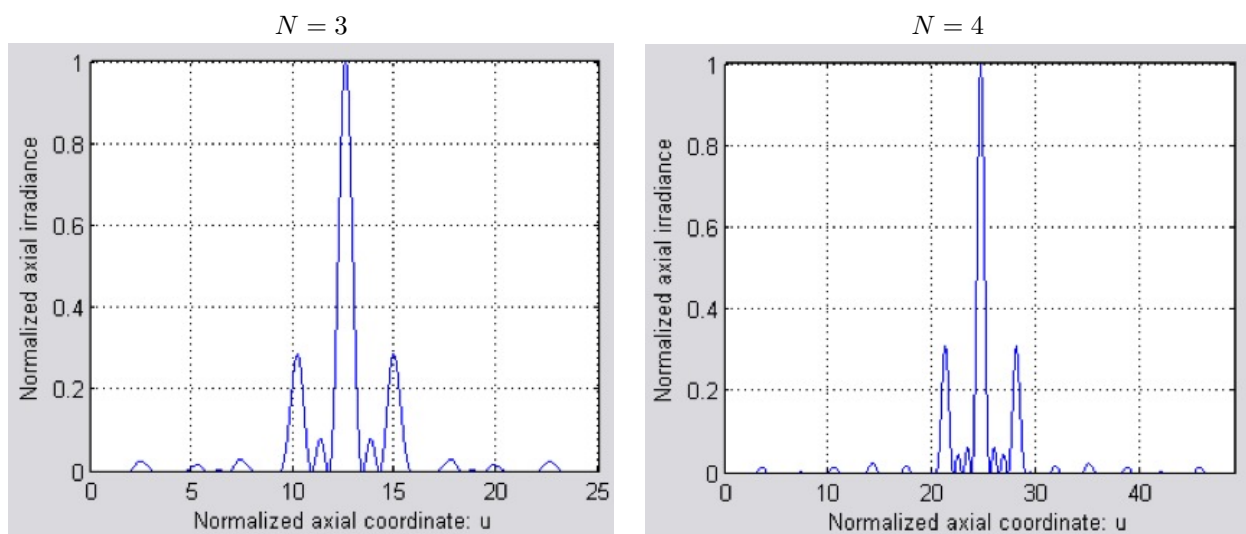

Figure 3.5: Normalized irradiance vs. the axial coordinate u obtained for a FZP $(S = 2)$ with different values of N.

generates three graphs: a bar chart of the pupil function (top right), the transmittance of the lens (lower right), and the normalized axial irradiance (left side).

- **zpdesigner.fig**: Contains graphic elements of zpdesigner.

- **gint.m, fint.m**: Auxiliary functions.

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