m-bonacci metamaterial multilayers: location of the zero-average index bandgap edges

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We examine quasiperiodic multilayers arranged in m-bonacci sequences, which combine ordinary positive-index materials and dispersive metamaterials with negative index in a certain frequency range. When the volume-averaged refractive index of the nonperiodic multilayer equals zero, the structure does not propagate light radiation and exhibits a forbidden band. We identify some analytical expressions to determine the upper and lower limits of the above zero-average refractive-index bandgap. We recognize that these limits are not explicitly dependent on the geometrical parameters of the stack of layers. © 2009 Optical Society of America

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Photonic crystals (PCs) allow propagation of electromagnetic waves in certain frequency bands but not in others [1], their essential feature being the periodic arrangement of materials showing high-contrast electromagnetic properties. The recent advent of metamaterials (MMs) has led to PCs that show impressive optical properties [2].

In this Letter we are interested in zero-$\bar{n}$ band gaps that appear in 1D PCs combining ordinary materials (with positive refractive index) and MMs (with negative refractive index) at frequencies where the volume average of the refractive index is zero [3]. Since these photonic bandgaps (PBGs) are not based on the standard Bragg interference mechanism, they are scale-length invariant and very robust against disorder. Experimental verifications of zero-$\bar{n}$ PBGs have been reported for 1D multilayers [4] and binary PC superlattices [5]. Apart from perfectly periodic PCs, deterministic aperiodic 1D structures also exhibit zero-$\bar{n}$ PBGs [6,7].

The complex variation of the shape and width of the zero-$\bar{n}$ gap with both the constitutive and geometrical parameters has been explored only for periodic multilayers [8]. It has been shown that the zero-$\bar{n}$ bandgap edges are approximately ruled by the following analytical expressions:

$$\bar{\mu}=0, \quad (\omega/c)^2\bar{\epsilon}+k_x^2\overline{\mu^{-1}}=0 \quad \text{(TE)}, \tag{1}$$

$$\bar{\epsilon}=0, \quad (\omega/c)^2\bar{\mu}+k_x^2\overline{\epsilon^{-1}}=0 \quad \text{(TM)}, \tag{2}$$

where k_x is the wave-vector component along the layers and $\bar{\epsilon}$, $\bar{\mu}$, $\overline{\epsilon^{-1}}$, and $\overline{\mu^{-1}}$ are the volume average of the dielectric permittivity, the magnetic permeability, and their inverses, respectively, in a period of the structure. Note that the above equations become

$$\bar{\mu}=0, \quad \bar{\epsilon}=0, \tag{3}$$

at normal incidence ($k_x=0$) for both polarizations. We recognize that these expressions are written in terms of quantities that do not explicitly depend on any geometrical parameter of the multilayer structure. Accordingly, their applicability could be more general.

The purpose of this Letter is to explore the validity of expressions (1) and (2) as good estimations for the zero-$\bar{n}$ bandgap edges for quasiperiodic multilayers. We would like to emphasize that quasiperiodic order has attracted considerable interest recently. From a theoretical point of view, it is considered a suitable theoretical model to describe the conceptual transition from randomness to periodic order. Besides, from a practical point of view, there is evidence that deterministically ordered aperiodic structures may offer interesting possibilities for technological applications [9]. Here we consider quasiperiodic MM stacks arranged in m-bonacci sequences, with $m=2$ (Fibonacci) and $m=3$ (Tribonacci). In contrast to periodic multilayers, in the quasiperiodic case the volume averages involved in Eqs. (1) and (2) depend on the number of fundamental building blocks of the m-bonacci sequence. This fact gives rise to a new variable, the generation level S of the structure.

Quasiperiodic binary multilayers based on the Fibonacci sequence are constructed following the recursive relation $D_S=\{D_{S-1},D_{S-2}\}$ for $S>2$, $D_1=\{A\}$, and $D_2=\{AB\}$. In this way, $D_3=\{ABA\}$, $D_4=\{ABAAB\}$, and so on. Note that the sequence at any generation level S contains $N_A=F_S$ A-layers and $N_B=F_{S-1}$ B-layers, where F_j, $j=0,1,2,\ldots$, is a Fibonacci number resulting from the sum of the two preceding numbers, $\{F_j\}=\{0,1,1,2,3,5,8,13,\ldots\}$. Tribonacci numbers T_j, $j=0,1,2,\ldots$ are like Fibonacci numbers, but the sequence starts with three predetermined terms. Besides each term is the sum of the preceding three ones, $\{T_j\}=\{0,1,1,2,4,7,13,24,\ldots\}$. For the construction of the quasiperiodic binary multilayers based on the Tribonacci sequence we use the recursive relation $D_S=\{D_{S-1},D_{S-2},D_{S-3}\}$ for $S>3$, with $D_1=\{A\}$, $D_2=\{AB\}$, and $D_3=\{ABAB\}$. Note that the sequence at an arbitrary generation level S contains $N_A=T_S$ A-layers and $N_B=T_{S-1}+T_{S-2}=T_{S+1}-T_S$ B-layers. Note that Fibonacci stacks of MMs and conventional dielectric materials were proposed previously [6,7], although their zero-$\bar{n}$ band limits were never discussed. Additionally, to the best of our knowledge, the aperiodic Tribonacci sequence has not been considered so far.

In our simulations A denotes a dispersive MM layer with effective constitutive parameters given by [3]

$$\epsilon_A(\nu) = 1 + \frac{5^2}{\nu^2 - 0.9^2} + \frac{10^2}{\nu^2 - 11.5^2}, \qquad (4)$$

$$\mu_A(\nu) = 1 - \frac{3^2}{\nu^2 - 0.902^2}, \qquad (5)$$

(ν in gigahertz), whereas B represents an air layer. The frequency variation of the effective parameters ϵ_A and μ_A is depicted in Fig. 1(a). The widths of the MM and air layers are $d_A=6$ mm and $d_B=12$ mm, respectively.

The photonic spectrum of the Fibonacci and Tribonacci 1D PCs at normal incidence (i.e., $k_x=0$) and generation levels from 2 to 11 are shown in Fig. 1. The calculation was carried out using the standard transfer-matrix method. The gray level indicates the value of the reflection coefficient R of these structures; white regions correspond to $R=1$, whereas black regions correspond to $R=0$. In the above systems, the average $\bar{\Phi}_S$ of a physical magnitude Φ for the generation level S is defined as

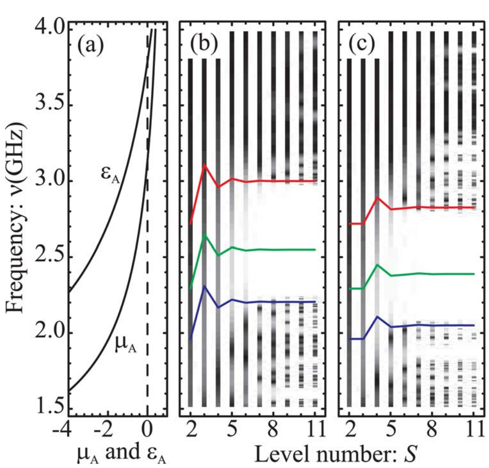

Fig. 1. (Color online) (a) Frequency behavior of the constitutive parameters corresponding to the MM layer. (b) Fibonacci and (c) Tribonacci photonic spectra at normal incidence for different generation levels.

$$\bar{\Phi}_S = \frac{\tau_S \Phi_A d_A + \Phi_B d_B}{\tau_S d_A + d_B}, \qquad (6)$$

where $\tau_S=N_A/N_B$. When $S\to\infty$, τ_S tends to the value $\tau = 1.61803$, the golden mean for the Fibonacci sequence. Similarly, $\tau = 1.19149$ for the Tribonacci sequence.

We observe that both kinds of quasiperiodic structures exhibit a forbidden bandgap in the region where the refractive index of the MM layer is negative. If we identify Φ with n in Eq. (6) and we consider $S\to\infty$, we conclude that they certainly correspond to zero-$\bar{n}$ gaps, whose central frequency is $\nu = 2.547$ GHz (for Fibonacci) and $\nu = 2.385$ GHz (for Tribonacci).

First we check that Eq. (3) also applies to determine the zero-$\bar{n}$ gap limits in the quasiperiodic case. To this end, we show in Fig. 1 the curves corresponding to $\bar{n}_S=0$ (middle curve), $\bar{\epsilon}_S=0$ (top curve) and $\bar{\mu}_S=0$ (bottom curve). It is interesting to note that the forbidden band is well defined only at slightly high generation levels, i.e., when the quasiperiodic multilayer can be regarded as an effective homogeneous material. For the Fibonacci multilayer, the requirements in Eq. (3) ($\bar{\epsilon}_S=0$ and $\bar{\mu}_S=0$), when $S\to\infty$, are met at frequencies 3.002 and 2.200 GHz, respectively. On the other hand, we numerically find that the zero-$\bar{n}$ gap edges are practically stabilized at the frequencies 2.973 and 2.240 GHz. Thus we conclude that in this aperiodic architecture, the approximate conditions given by Eq. (3) provide, as in the periodic case, a good estimation of the zero-$\bar{n}$ gap edges, the relative errors being 0.96% (upper limit) and 1.84% (lower limit). Similarly, for the Tribonacci multilayer, the conditions $\bar{\epsilon}_S=0$ and $\bar{\mu}_S=0$ for $S\to\infty$ are fulfilled at frequencies 2.826 and 2.043 GHz, respectively, whereas the transfer-matrix method supplies the values 2.790 and 2.093 GHz, respectively. Despite using a rather different stack of layers, we highlight the fact that Eq. (3) still provides a good estimation of the zero-$\bar{n}$ gap edges. In the latter case the relative errors are 1.28% (upper limit) and 2.42% (lower limit).

In Fig. 2 we plot the dependence of the photonic spectra of the above aperiodic structures on the angle of incidence, for TE and TM polarizations. We have considered two different situations, the Fibonacci multilayer $S=10$ and the Tribonacci structure with $S=8$. In this way, we compare two aperiodic structures with nearly the same number of layers, 89 in the Fibonacci case ($N_A=55$ and $N_B=34$) and 81 in the Tribonacci one ($N_A=44$ and $N_B=37$). We observe that for oblique propagation, Eqs. (1) and (2) also provide a very good estimation of the zero-$\bar{n}$ gap edges (dashed curves in Fig. 2). Note that in these equations the lower limit for TE polarization ($\bar{\mu}=0$) and the upper limit for TM polarization ($\bar{\epsilon}=0$) are independent of the incident angle.

Summarizing, we have analyzed the behavior of zero-$\bar{n}$ PBG edges in quasiperiodic multilayers based on m-bonacci sequences combining ordinary positive

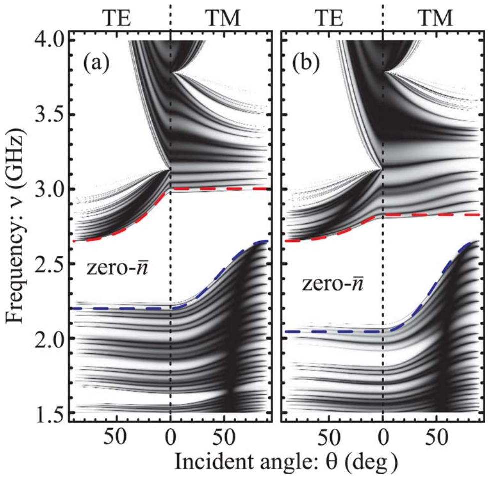

Fig. 2. (Color online) Photonic spectrum as a function of the incident angle for (a) the Fibonacci MM multilayer with level number $S=10$ and (b) the Tribonacci MM multilayer with level number $S=8$.

index materials and dispersive MMs. In this case, the sequence for the generation of the aperiodic configuration determines the central frequency of the zero-$\bar{n}$ gaps. The upper and lower limits of these bandgaps can be accurately approximated by analytical expressions, which are exactly the same as the ones that apply for periodic multilayers at the low-frequency limit. It cannot be otherwise, since the above band limits only involve the volume average of the constitutive parameters. This evidences the validity of this extension. Consequently, it is apparent that the above results may be generalized to other aperiodic multilayers, such as Thue–Morse, period doubling, or silver-mean lattices, among others.

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