Diffraction by fractal metallic supergratings

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Abstract: The reflectance of corrugated surfaces with a fractal distribution of grooves is investigated. Triadic and polyadic Cantor fractal distributions are considered, and the reflected intensity is compared with that of the corresponding periodic structure. The self-similarity property of the response is analyzed when varying the depth of the grooves and the lacunarity parameter. The results confirm that the response is self-similar for the whole range of depths considered, and this property is also maintained for all values of the lacunarity parameter.

OCIS codes: (050.1940) Diffraction; (050.1970) Diffractive optics

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1. Introduction

The electromagnetic diffraction from surfaces having rectangular profiles has been studied by many authors [1]-[7]. This kind of surfaces is of interest because they can be manufactured quite easily and they permit an accurate control of their parameters, making it possible to compare theoretical with experimental data [2]. Most of the investigations have been devoted to ideal gratings, i.e., those with strictly periodic, unlimited boundaries separating two media. In particular, the case of perfectly conducting materials have been studied by Andrewartha et *al*. [3] and Wirgin and Maradudin [4].

An increasing interest in the usage of light diffracted as a means of measuring surface microgeometry lead many efforts to study the direct problem deeply. To compare experimental with theoretical results, Maystre [8] developed a rigorous theory to study the reflected electromagnetic field from perfectly conducting finite gratings, that is, gratings having a finite number of grooves. Several methods for solving the scattering problem by grooves in a plane are mentioned in refs. [9] and [10].

Park et al. [7] studied the problem of diffraction of a *s*-polarized plane wave by equally spaced rectangular grooves in a perfectly conducting plane by means of a method using a modal representation for the field inside the grooves. In [11], a rigorous modal theory is presented, to solve the problem of light reflected from rough surfaces having a finite number of randomly distributed rectangular corrugations. With this model, the width of each groove and the distance between each pair of adjacent grooves can be chosen arbitrarily, for both polarization modes. The method presented in [11] was later used to study the superdirective property of passive antennas [12], and more recently, the same formalism was extended to deal with compound gratings –gratings comprising several grooves within each period–, and their properties have been studied [13, 14, 15].

In recent years the study of fractals has attracted the attention of researchers, encouraged by the fact that many physical phenomena, natural structures and statistical processes can be analyzed and described by using a fractal approach [16]. From a mathematical point of view the concept of fractal is associated with a geometrical object which i) is self-similar (i.e., the object is exactly or approximately similar to a part of itself) and, ii) has a fractional (or noninteger) dimension. In optics, diffractive fractal structures, ranging from simple one-dimensional (1D) objects [17, 18] to complex 2D systems [19, 20] have been extensively analyzed. Ledesma *et al.* [21] studied the diffraction properties of metallic fractal gratings with rectangular protuberances generated of the Cantor type, where the width and the depth of the corrugations is small

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compared with the incident wavelength. For this particular case, they showed that the spectral response of such structures exhibits self-similarity features.

In this paper we face the more general problem of obtaining the reflection properties of fractal structures with arbitrary widths and depths. We use the rigorous modal method presented in [11] which permits to evaluate the validity of the self-similarity property of the response when the thickness of the structure is increased. In addition, we focus our attention on fractal supergratings based on the generalized Cantor-set etched in metallic surfaces. We study the response to a specific design parameter, frequently used as a measure of the “texture” of fractal structures: the lacunarity.

This paper is organized as follows. In Section 2 we present the procedure we followed for the design of fractal metallic gratings. In Section 3 we summarize the fundamentals of the modal method employed to solve the diffraction problem by corrugated surfaces. In Section 4 we investigate the self-similarity property of triadic Cantor gratings, and compare their response with that of a perfectly periodic grating. We also show that the self-similar property is maintained when the depth of the grooves is increased. In Section 5, the lacunarity parameter is introduced in the structure, and the dependence of the response with this parameter is studied. Finally, concluding remarks are given in Section 6.

2. Triadic fractal metallic gratings design

One of the classic and most well known fractals is the triadic Cantor set. As shown in Fig. 1, it can be constructed by an iterative process. The first step ($S = 0$) is to take a segment of length a. The second step ($S = 1$) is to divide the segment in three equal parts of length $a/3$ and remove the central one. Then, on each of these segments this procedure is repeated, and so on. The Cantor-set is the set of segments remaining. In general, at stage S, there are 2^S segments of length $a/3^S$ with $2^S - 1$ gaps in between. In Fig. 1, only the four first stages are shown for clarity.

Based on this scheme we propose a fractal metallic grating which, as it is shown in Fig. 2(a), is defined by the parameter S. At the S-th stage presents 2^S grooves corresponding to the black regions of the Cantor set. Note that this structure can be interpreted as a quasiperiodic metallic grating which can be obtained by ”filling in” some grooves of a finite periodic grating as the one shown in Fig. 2(b). This distribution has $(3^S + 1)/2$ segments at stage S each one of width $a/3^S$, separated by gaps of the same length, so that the period of this finite structure is $2a/3^S$.

3. Modal theory

Consider a general metallic plane with L grooves of the same height h and widths c_l ($l = 1, 2, \ldots, L$) as shown in Fig. 2(a). A plane wave of wavelength λ is incident upon the surface from the region $y > 0$ forming an angle θ_0 with the y-axis, being the x,y-plane the plane of

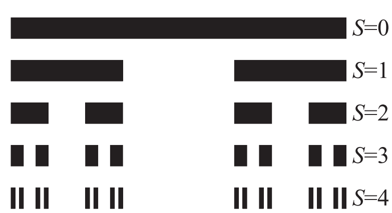

Fig. 1. Triadic Cantor set for the first levels of growth, S. The structure for $S = 0$ is the initiator and the one corresponding to $S = 1$ is the generator. Black regions correspond to the grooves etched in the fractal metallic grating (see Fig. 2)

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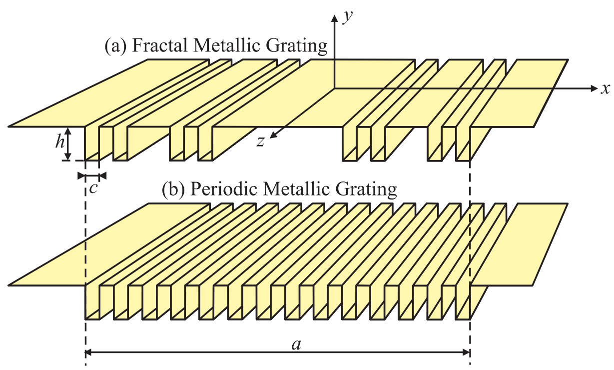

Fig. 2. (a) Fractal ($S = 3$) and (b) periodic metallic gratings.

incidence. Assuming an harmonic time dependence in the form $\exp(-i\omega t)$, where ω is the frequency of the incident light, Maxwell's equations are combined to get the Helmholtz equation that must satisfy the fields $\vec{E}$ and $\vec{H}$ everywhere. The vectorial problem can be separated into two independent scalar problems: s-polarization (electric field parallel to the grooves) and p-polarization (magnetic field parallel to the grooves). We call f^{μ} to the z-component of the electric (magnetic) field in the case of s- (p-) polarization ($\mu = s, p$). We separate the space into two regions (see Fig. 2): the region $y \geq 0$ (+), in which the scattered fields are represented by a continuous superposition of plane waves (Rayleigh expansion), and the region inside the grooves, $-h \leq y \leq 0$ (−), where the fields are represented using modal expansions.

In the upper region, $y \geq 0$, we express the total field, $f^{\mu}_{+}(x,y)$, as the sum of the following terms: the incident field,

$$f_{\rm inc}(x,y) = e^{i(\alpha_0 x - \beta_0 y)} \quad , \tag{1}$$

the speculary reflected field,

$$f^{\mu}_{\rm spec}(x,y) = (-1)^j e^{i(\alpha_0 x - \beta_0 y)} \;\; ; \quad j = \begin{cases} 1 & \text{for } s\text{-polarization} \\ \\ 0 & \text{for } p\text{-polarization} \end{cases} \quad , \tag{2}$$

and the scattered field,

$$f^{\mu}_{\rm scatt}(x,y) = \int_{-\infty}^{\infty} R^{\mu}(\alpha)\, e^{i(\alpha x + \beta y)} d\alpha \;\; . \tag{3}$$

The parameters $\alpha = k_0 \sin\theta$ and $\beta = \sqrt{{k_0}^2 - \alpha^2}$ are the x- and y-components of the wavevector ($k_0 = \omega/c = 2\pi/\lambda$) in the θ direction, respectively. To obtain the Rayleigh amplitudes, $R^{\mu}(\alpha)$, corresponding to the scattered field, we must consider the fields inside the grooves which are expressed in terms of the corresponding modal functions for each polarization. For the l-th groove we have:

$$f^{s}_{-}(x,y) = \sum_{m=1}^{\infty} a_{m,l} \sin\left[\mu_{m,l}(y+h)\right] \sin\left[\frac{m\pi}{c_l}(x - x_l)\right] \; , \tag{4}$$

and

$$f^{p}_{-}(x,y) = \sum_{m=0}^{\infty} b_{m,l} \cos\left[\mu_{m,l}(y+h)\right] \cos\left[\frac{m\pi}{c_l}(x - x_l)\right] \; . \tag{5}$$

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where

$$\mu_{m,l} = \begin{cases} \sqrt{k_0{}^2 - (\frac{m\pi}{c_l})^2} & \text{if } k_0{}^2 > (\frac{m\pi}{c_l})^2 \\ i\sqrt{(\frac{m\pi}{c_l})^2 - k_0{}^2} & \text{if } k_0{}^2 < (\frac{m\pi}{c_l})^2 \end{cases}, \tag{6}$$

and $a_{m,l}$ and $b_{m,l}$ are the modal amplitudes corresponding to s- and p- polarization, respectively.

Matching the fields at the plane $y = 0$ we obtain a system of coupled equations for each polarization mode, which are projected in convenient bases to drop the x dependence. The projected equations are then combined to yield an integral equation for the Rayleigh amplitudes $R^s(\alpha)$ and $R^p(\alpha)$, for s and p polarization, respectively. Once these amplitudes are found, the normalized intensity of the reflected field in the θ direction is calculated by $I(\alpha) = |R^{\mu}(\alpha)|^2$.

4. Self-similarity in fractal metallic gratings

Following the approach presented in the previous section we computed the angular reflected intensity of metallic fractal gratings. The result is shown in Fig. 3 where we compare the angular response of fractal surfaces for different steps S (top plots), with those of the corresponding periodic grating (bottom plots). The width of the grooves in each case is $c/a = 1/3^S$, the depth is $h/a = 0.00077125$. The incident wave is s-polarized, with a wavelength $\lambda/a = 0.00617$, and $\theta_0 = 0°$. Figures 3(a), 3(b) and 3(c) correspond to different stages of the fractal, and then each one of the surfaces have 4 ($S = 2$), 8 ($S = 3$), and 16 ($S = 4$) grooves, respectively. The corresponding periodic surfaces have 5, 14, and 41 grooves, respectively. It can be observed that the position of the main maxima, which is given by the periodicity of the structure, is the same for the periodic as well as for the fractal structures. Taking into account that the period of

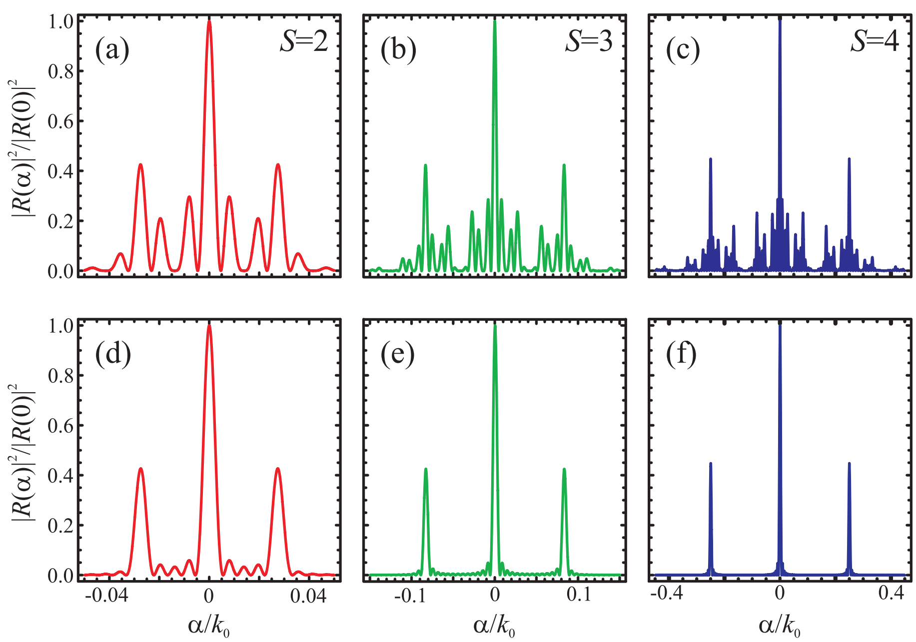

Fig. 3. Angular reflected response of a metallic plane with grooves with fractal distribution for different steps S (top plots), and with the corresponding periodic distribution as shown in Fig. 1 (bottom plots).

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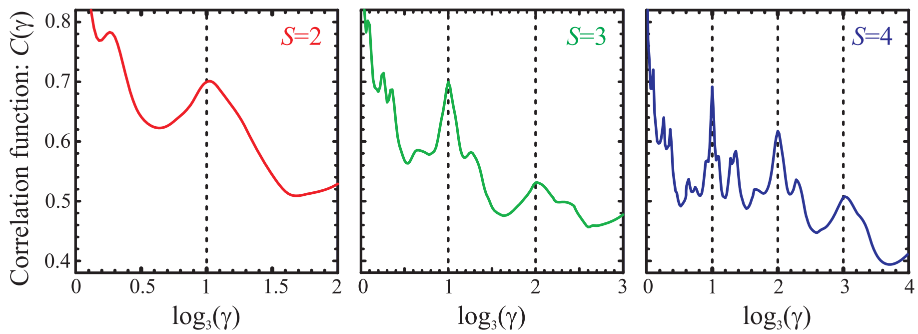

Fig. 4. Correlation coefficient as a function of $\log_3(\gamma)$, for the same three fractal structures considered in Fig. 3.

the structure is $2a/3^S$, the directions α/k_0 at which the reflectance is maximum are given by:

$$\frac{\alpha}{k_0} = \frac{\alpha_0}{k_0} + n\,\frac{\lambda}{a}\,\frac{3^S}{2}\ , \tag{7}$$

with n integer. For the parameters of Fig. 3, the first maximum ($n=1$) for $S=2$ should be at $\alpha/k_0 = 0.027765$, for $S=3$ at $\alpha/k_0 = 0.083295$, and for $S=4$ at $\alpha/k_0 = 0.249885$. The positions of the maxima agree very well with these calculated values.

An assesment of the self-similarity of the system response can be obtained by means of the correlation between the values of the scattered intensities produced by fractal gratings of different scales. This function is defined as [17]:

$$C(\gamma) = \frac{\int (I(\alpha)-\bar{I})\,(I(\alpha/\gamma)-\bar{I}_\gamma)\,d\alpha}{\left[\int (I(\alpha)-\bar{I})^2\,d\alpha\ \int (I(\alpha/\gamma)-\bar{I}_\gamma)^2\,d\alpha\right]^{1/2}}\ , \tag{8}$$

where γ is a scale factor, and $\bar{I}$ and $\bar{I}_\gamma$ are the mean values of the function and of its scaled version, respectively. In Fig. 4 we plot the self-similarity coefficient as a function of $\log_3(\gamma)$, for the same three fractal structures considered in Fig. 3. This coefficient exhibits local maxima at 3^m, being m a natural number, and the number of peaks increases with the stage S of the fractal.

The modal method used for the calculation of the reflected fields is efficient even for very deep structures, and this advantage allows us to investigate the dependence of the fractal properties of the system response when the thickness of the metal plate is increased. The Rayleigh method employed in [21] is an approximate method, and is only valid for small heights of the corrugations. Figure 5 are grey-scale maps of the system response (in dBs) as a function of α and of the depth of the grooves, for $S=2$ [Fig. 5(a)] and for $S=3$ [Fig. 5(b)]. The parameters of the structures and of the incident wave are the same as those in Fig. 3. It can be observed that as the depth of the grooves is increased, the response of the fractal system keeps the same characteristics, with its maxima and minima located in the same positions as for the small depth case [Figs. 3(a) and 3(b)], what corresponds to peaks at the same scale factors γ in the self-similarity coefficient, as those found for the previous case [Figs. 4(a) and 4(b)]. It can be seen that for both cases ($S=2$ and $S=3$) the reflected intensity is a periodic function of h/a as predicted by Eqs. (4) and (5). A rough estimate of this period can be done assuming that the fundamental mode in each groove is the only relevant mode, and then the period $\Delta h/a \approx \pi/\mu_1 = \{\sqrt{(2a/\lambda)^2-1}\}^{-1} \approx 0.003$, as observed in Figs. 5(a) and 5(b).

In order to perform an exhaustive analysis, we have also computed the reflected intensities when the incident wave is p-polarized. We have found that the self similarity behavior is pre-

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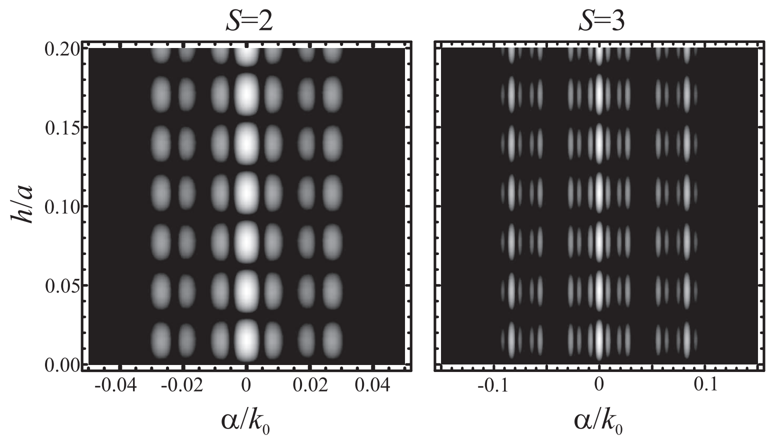

Fig. 5. Grey-scale maps of the system angular reflected intensity (in dBs) as a function of α and of the depth of the grooves h/a. (a) $S = 2$; (b) $S = 3$.

served for this polarization, as well as the periodic behavior with the depth of the grooves. Taking into account the fundamental mode, the period in h/a for the *p*-case can be estimated as $\lambda/2a$, which for the parameters studied in this paper gives roughly the same value as for the *s*-case (these results have not been included for brevity).

5. Lacunarity in fractal metallic supergratings

The construction of the triadic Cantor set may be generalized by dividing each segment into more than three parts yielding the symmetrical polyadic Cantor fractal sets [16, 22]. The initiator (step $S = 0$) is again a straight line segment of unit length. At step $S = 1$ the initiator is replaced by N nonoverlapping copies of the initiator, each one scaled by a factor $\gamma < 1$. For even N, as shown in Fig. 6, one half of the copies are placed to the left of the interval and the other half to its right, each copy being separated by a fixed distance ε. For odd N, not shown in Fig. 6, one copy lies centered in the interval and the rest are distributed as for even N, that is, [N/2] copies are placed to the left of the interval and the other [N/2] copies to its right, where [N/2] is the greatest integer less than or equal to N/2. At each step of the construction, the generation process is repeated over and over again for each segment in the previous step.

The construction parameter ε is arbitrary and it can be associated to the lacunarity [16], which may be defined as the deviation of a fractal from translational invariance [23]. Translational invariance is highly scale dependent because heterogeneous sets at small scales can be homogeneous on larger scales or vice versa. Actually, the lacunarity is a scale-dependent measure of the heterogeneity (or texture) of an object, whether or not it is fractal. For the purposes of this paper it is sufficient to consider ε as an indication of the lacunarity of the generalized Cantor set, as shown in Fig. 6 [22, 24].

Symmetrical polyadic Cantor fractals are characterized by the number of self-similar copies N, the scaling factor γ, and the lacunarity parameter ε. The last two parameters must satisfy certain constraints to avoid overlapping between the copies. The maximum value of the scaling factor depends on the value of N, such that $0 < \gamma < \gamma_{\max} = 1/\mathrm{N}$. For each value of N and γ, there are two extreme values for ε. One is $\varepsilon_{\min} = 0$, for which the highest lacunar fractal is obtained, that is, one with the largest possible gap. For even N, the central gap has a width of $1 - \mathrm{N}\gamma$, and for odd N, both large gaps surrounding the central well have a width $(1 - \mathrm{N}\gamma)/2$.

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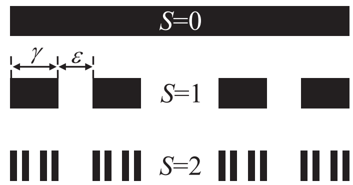

Fig. 6. First steps of the development of polyadic, N=4, symmetrical generalized Cantor sets. The definitions of the scale factor γ and of the lacunarity parameter ε characterizing polyadic Cantor sets are also shown.

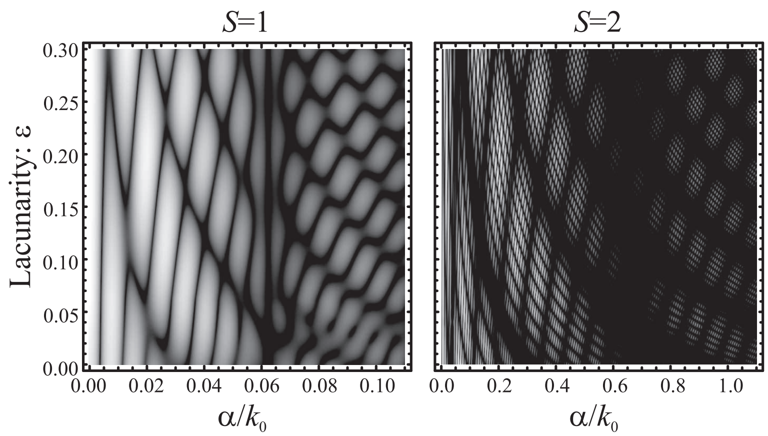

Fig. 7. Twist plots, that is, gray-scale representations of the reflected intensity (in dB) as a function of α/k_0 and of the lacunarity parameter ε for the polyadic Cantor prefractal distributions for (a) $S=1$ and (b) $S=2$, for N=4 and $\gamma=0.1$.

The other extreme value is

$$\varepsilon_{\max} = \begin{cases} \frac{1-\mathrm{N}\gamma}{\mathrm{N}-2} & \text{even N} \\ \frac{1-\mathrm{N}\gamma}{\mathrm{N}-3} & \text{odd N} \end{cases}, \tag{9}$$

where for even (odd) N two (three) wells are joined together in the center and the central gap is missing. The width of the N-2 gaps in this case is equal to $\varepsilon_{\max}$. Thus the corresponding lacunarity is smaller than that for $\varepsilon=0$, but not the smallest one, which is obtained for the most regular distribution, where the gaps and wells have the same width at the first step ($S=1$) given by

$$\varepsilon_{\mathrm{reg}} = \frac{1-\mathrm{N}\gamma}{\mathrm{N}-1}\ . \tag{10}$$

Note that $0<\varepsilon_{\mathrm{reg}}<\varepsilon_{\max}$.

To show that the self-similar behavior of the reflected intensity for polyadic Cantor supergratings is retained even when the lacunarity parameter is varied, twist plots [22] may be used. These plots represent the intensity as a function of the normalized x-component of the wavevector (α/k_0) and the lacunarity parameter ε. Figure 7 shows twist plots for tetradic (N=4) Cantor supergratings for $S=1$ (a) and $S=2$ (b). In these plots a logarithmic gray scale was used for the reflection coefficient, from black for zero values to white for the maximum value equal to unity.

It can be observed that the re-scaled reflected intensity at step $S=1$ forms an envelope for the (unscaled) reflected intensity at step $S=2$, showing that the response is self-similar for

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any value of the lacunarity parameter ε. Notice that in each plot there is a vertical dark line (no reflected intensity) that corresponds to $\alpha/k_0 = 0.0617$ and 0.617 for $S = 1$ and $S = 2$, respectively. It is remarkable that this null does not depend on the value of ε. The origin of these bands can be understood as diffraction minima, produced by the diffraction by a single groove. The diffraction minima produced by a single groove are in the observation directions given by

$$\frac{\alpha}{k_0} = m\,\frac{\lambda}{c}\ , \qquad \text{for } m \text{ integer.} \tag{11}$$

For the value of γ considered in these figures, $c/a = 0.1$ ($S = 1$) and $c/a = 0.01$ ($S = 2$), in which case eq. (11) gives $\alpha/k_0 = 0.0617$ ($S = 1$) and 0.617 ($S = 2$), exactly the positions at which the vertical nulls are found. The other nulls in these plots can be understood as multiple cross-interferences between different grooves of the fractal reflecting structure.

6. Conclusion

The reflected response of corrugated surfaces with a fractal distribution of grooves have been investigated. Triadic and polyadic Cantor fractal distributions were considered. The scattering problem of a plane wave by a fractal reflecting structure was solved by means of the modal method. We compared the results for the triadic Cantor structure with those for the corresponding periodic structures, and showed that the position of the main maxima is the same for both types of corrugated surfaces. It was shown that when the depth of the grooves is increased, the self-similarity property of the response is maintained. The results also evidence that this property is also valid when the lacunarity parameter of the polyadic Cantor structure is varied.

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