Analysis of 3-D Integral Imaging Displays Using the Wigner Distribution

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***Abstract*—Integral imaging is a promising technology for 3-D TV and 3-D display. In this paper, a theoretical analysis of 3-D integral imaging systems is performed in the frame of the Wigner distribution formalism. It is shown that the entire intensity distribution in the pick-up image plane of these systems can be obtained from a single 2-D Wigner distribution function of a single lenslet pupil. This result reveals the Wigner distribution function as a powerful tool for analysis of 3-D integral imaging systems with different pupil functions. As an example, the extension of the depth of field of an integral imaging system with lenslets having amplitude modulation (central obscuration) is proposed.**

***Index Terms*—Multiple perspective imaging, three-dimensional (3-D) image acquisition, 3-D image processing.**

I. Introduction

INTEGRAL IMAGING is a 3-D imaging technique that works with naked eyes providing autostereoscopic images from a continuous viewpoint [1]–[5]. It is considered to be a promising approach to 3-D TV and 3-D display systems [6], [7]. In an integral imaging (II) system, a 2-D collection of elemental images of a 3-D object is generated by a microlens array, and recorded in a sensor such as a CCD. Each elemental image has a different perspective of the 3-D object. The recorded 2-D elemental images are displayed by an optical device, such as a liquid-crystal display (LCD), placed in front of another microlens array to reconstruct the 3-D image. Recently, many important theoretical and experimental contributions on integral imaging have been reported [8]–[20].

On the other hand, phase-space functions like the Ambiguity function and the Wigner distribution function (WDF) have been proven to be excellent tools for the analysis and synthesis of optical imaging systems [21]–[28]. However, to the best of our knowledge these formalisms have never been applied to II systems. In this paper, an analysis of the performance of II is performed within the formalism of the WDF. In Section II, the definition of the WDF and some properties that are relevant for our purposes are reviewed. In Section III, the relationship between the intensity at the image plane of a general II system and the WDF of the microlenses pupils is investigated. In particular we focus our attention on separable pupils to look for a method for computing the intensity given by a general 3-D object using a single 2-D WDF. Since the depth of field (DOF) of the system is one of the main performance concerns of II systems, our analysis is performed to take into account this parameter explicitly. Finally, in Section IV as a practical application of the usefulness of the WDF in the analysis of II, a simple approach to increase the DOF in II is proposed by use of separable amplitude masks for the microlenses.

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II. Wigner Distribution Function. Definition and Some Properties

The WDF of an n-dimensional complex-valued function $f(\mathbf{x})$ is a joint representation of this function in the space and spatial frequency-domain $(\mathbf{x}, \boldsymbol{\nu})$. It is defined as

$$W_f(\mathbf{x}, \boldsymbol{\nu}) = \int_{-\infty}^{+\infty} f\left(\mathbf{x} + \frac{\mathbf{x}'}{2}\right) f^*\left(\mathbf{x} - \frac{\mathbf{x}'}{2}\right) \exp(-i2\pi\mathbf{x}'\boldsymbol{\nu}) d\mathbf{x}', \tag{1}$$

In (1) the symbol $*$ denotes the complex conjugate. Directly from the above definition the following properties of the WDF can be obtained:

1) *Dual definition:* If $F(\boldsymbol{\nu})$ is the Fourier transform of $f(\mathbf{x})$ i.e.,

$$F(\boldsymbol{\nu}) = \int_{-\infty}^{+\infty} f(\mathbf{x}) \exp(-i2\pi\mathbf{x}\boldsymbol{\nu}) d\mathbf{x}. \tag{2}$$

The WDF of $f(\mathbf{x})$ can also be defined as

$$W_f(\mathbf{x}, \boldsymbol{\nu}) = \int_{-\infty}^{+\infty} F\left(\boldsymbol{\nu} + \frac{\boldsymbol{\nu}'}{2}\right) F^*\left(\boldsymbol{\nu} - \frac{\boldsymbol{\nu}'}{2}\right) \exp(\mathrm{i}2\pi\mathbf{x}\boldsymbol{\nu}') d\boldsymbol{\nu}' \tag{3}$$

2) *Inversion:* By noting that (1) is a Fourier transform relation that can be inverted, it can be easily shown that the original function can be recovered from its WDF up to a complex constant

$$f(\mathbf{x}) = \frac{1}{f^*(0)} \int_{-\infty}^{+\infty} W_f(\mathbf{x}/2, \boldsymbol{\nu}) \exp(i2\pi\mathbf{x}\boldsymbol{\nu}) d\boldsymbol{\nu} \tag{4}$$

3) *Finite support:* In the case of space-limited signals, the WDF is zero out of the range of signal definition. The same property applies to the spatial-frequency domain.

4) *Realness:* The WDF is a real function (this property is a consequence of the Hermiticity property of the product inside the integral in (1)). However, it is not always positive.
5) *Space and frequency shift:* Shifts in space or frequency domains of the original function give corresponding shifts in the WDF

$$g(\mathbf{x}) = f(\mathbf{x}-\mathbf{x_0}) \quad \rightarrow W_g(\mathbf{x},\boldsymbol{\nu}) = W_f(\mathbf{x}-\mathbf{x_0},\boldsymbol{\nu}) \quad (5)$$
$$G(\boldsymbol{\nu}) = F(\boldsymbol{\nu}-\boldsymbol{\nu_0}) \quad \rightarrow W_g(\mathbf{x},\boldsymbol{\nu}) = W_f(\mathbf{x},\boldsymbol{\nu}-\boldsymbol{\nu_0}) \quad (6)$$

6) *Interference:* The WDF of the sum of two signals $f(\mathbf{x}) + g(\mathbf{x})$ is given by

$$W_{f+g}(\mathbf{x},\boldsymbol{\nu}) = W_f(\mathbf{x},\boldsymbol{\nu}) + W_g(\mathbf{x},\boldsymbol{\nu}) + 2\mathrm{Re}\left[W_{f,g}(\mathbf{x},\boldsymbol{\nu})\right] \quad (7)$$

where Re is the real-part-taking operation. The last term in (7), is a "cross" or interference term between the WDF of the original functions which is known as the cross-WDF corresponding to $f(\mathbf{x})$ and $g(\mathbf{x})$.
7) *Marginals:* Some integrals of the WDF have a clear physical meaning. For instance, the integrals over the space and the spatial frequency variable represent the intensity of the frequency spectrum and the intensity of the signal, respectively

$$\int_{-\infty}^{+\infty} W_f(\mathbf{x},\boldsymbol{\nu}) d\boldsymbol{\nu} = |f(\mathbf{x})|^2, \quad (8)$$
$$\int_{-\infty}^{+\infty} W_f(\mathbf{x},\boldsymbol{\nu}) d\mathbf{x} = |F(\boldsymbol{\nu})|^2. \quad (9)$$

Additional properties of WDF can be found elsewhere [21], [22].

III. 3-D II in Terms of the Wigner Distribution

Having revisited the definition and some useful properties of the WDF, we now turn to II to express their imaging properties within this formalism. In II, lenslet arrays are used as depicted in Fig. 1. A set of elemental images of a 3-D object (i.e., direction and intensity information of the spatially sampled rays coming from the object) are obtained by use of a lenslet array and a 2-D image sensor such as a CCD array [Fig. 1(a)]. To reconstruct a 3-D image of the object, the set of 2-D elemental images are displayed in front of a lenslet array using a 2-D display panel, such as a LCD panel [Fig. 1(b)]. The rays coming from elemental images converge to form a 3-D real image through the lenslet array. The reconstructed 3-D image is a pseudoscopic (depth-reversed) real image of the object. To convert the pseudoscopic image to an orthoscopic image, a process to rotate every elemental image by 180 deg around its own center optic axis may be used [18]. The orthoscopic image becomes a virtual image by this P/O conversion process.

Let us now consider in detail the pick-up setup as shown in Fig. 1(a). The light reflected by the surface of the object when illuminated by a spatially incoherent beam is collected by the microlens array which forms a set of 2-D elemental images in the image plane. Due to the spatial distribution of the microlenses each elemental image has a different perspective of the object. In the pick-up plane $\mathbf{x}' = (\mathrm{x}', \mathrm{y}')$, the intensity distribution is obtained as the superposition of the diffraction spots produced by any point of the object surface. Using scalar diffraction theory it has been shown [14] that this intensity is given by

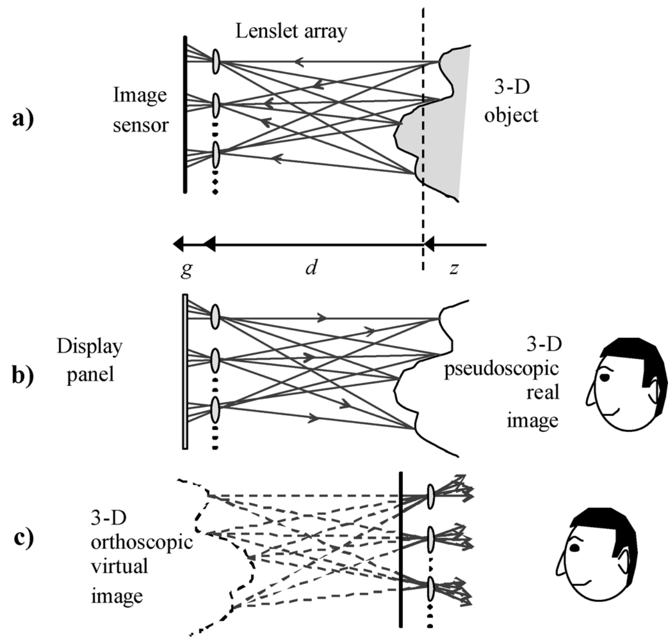

Fig. 1. 3-D integral imaging. (a) Pickup (the dashed line represents the reference object). (b) Real image display. (c) Virtual image display. (Color version available online at http://ieeexplore.ieee.org.)

$$I(\mathbf{x}') = \int_{\mathrm{R}^2} R(\mathbf{x}) H\left[\mathbf{x}'; \mathbf{x}, z = f(\mathbf{x})\right] d^2\mathbf{x} \quad (10)$$

Where the object is defined by two functions, namely, its surface, $f(\mathbf{x}) - z = 0$ and its reflectivity $R(\mathbf{x})$; and the function $H(\mathbf{x}'; \mathbf{x}, z)$ is given by

$$H(\mathbf{x}'; \mathbf{x}, z) = H_o(\mathbf{x}'; z) \otimes \sum_{\mathbf{m}} \delta\left\{\mathbf{x}' - [\mathbf{m}p(1-M_z) - M_z\mathbf{x}]\right\} \quad (11)$$

In this equation, $\mathbf{m} = (m, n)$ accounts for the microlenses indexes in the (x, y) directions, p is the pitch of the square lattice where the lenslets are packed, and $M_z = -g/(d+z)$ is the lateral magnification. The symbol $\otimes$ denotes the 2-D convolution product, and $\delta(\bullet)$ is the Dirac delta function. The function $H_{\mathrm{o}}(\mathbf{x}'; z)$ is the squared modulus of the Fourier transform of the generalized pupil function that takes into account the microlens pupil function $\mathrm{p}(\mathbf{x_o})$ and the phase modulation due to defocus [14]

$$H_o(\mathbf{x}'; z) = \left| \int_{\mathrm{R}^2} p(\mathbf{x_o}) \exp\left(-i\frac{\pi z}{\lambda d(d+z)}|\mathbf{x_o}|^2\right) \times \exp\left(-i2\pi\mathbf{x_o}\frac{\mathbf{x}'}{\lambda g}\right) d^2\mathbf{x_o} \right|^2.$$
$$= \int_{\mathrm{R}^2}\int_{\mathrm{R}^2} p(\mathbf{x_o}) p^*(\mathbf{x}'_\mathbf{o}) \times \exp\left(-i\frac{\pi z}{\lambda d(d+z)}\left(|\mathbf{x_o}|^2 - |\mathbf{x}'_\mathbf{o}|^2\right)\right) \times \exp\left(-i2\pi\frac{\mathbf{x}'(\mathbf{x_o}-\mathbf{x}'_\mathbf{o})}{\lambda g}\right) d^2\mathbf{x_o} d^2\mathbf{x}'_\mathbf{o}. \quad (12)$$

If we perform the change of variables $\zeta = (\mathbf{x_o} + \mathbf{x'_o})/2$; $\zeta' = \mathbf{x_o} - \mathbf{x'_o}$, the last equation can be rewritten in terms of the WDF of the pupil function. In fact, from the definition of the WDF [see (1)] we obtain

$$\begin{aligned} H_o(\mathbf{x}'; z) = &\int_{\mathsf{R}^2}\int_{\mathsf{R}^2} p(\zeta + \zeta'/2)p^*(\zeta - \zeta'/2) \\ &\times \exp\left(-i\frac{\pi z}{\lambda d(d+z)}2\zeta\zeta'\right) \\ &\times \exp\left(-i2\pi\frac{\mathbf{x}'\zeta'}{\lambda g}\right) d\zeta d\zeta' \\ = &\int_{\mathsf{R}^2} W_p\left(\zeta; \zeta\frac{z}{\lambda d(d+z)} + \frac{\mathbf{x}'}{\lambda g}\right) d\zeta \end{aligned} \quad (13)$$

Note that the WDF inside the integral in (13) is a 4-D function of the 2-D pupil function $p(x_o, y_o)$. Now, assuming that in the II system each lenslet element is square shaped, then, in this case, the pupil function is separable in the form $p(\mathbf{x}) = p(\mathrm{x}, \mathrm{y}) = p(\mathrm{x})p(\mathrm{y}) = \mathrm{rect}(x/\mathrm{a})\mathrm{rect}(y/\mathrm{a})$. Therefore, the function $H_\mathrm{o}(\mathbf{x}'; z)$ is also separable and can be expressed as

$$\begin{aligned} H_o(\mathbf{x}'; z) = &\int \mathrm{W_p}\left(\zeta; \zeta\frac{z}{\lambda d(d+z)} + \frac{x'}{\lambda g}\right) d\zeta \\ &\times \int \mathrm{W_p}\left(\eta; \eta\frac{z}{\lambda d(d+z)} + \frac{y'}{\lambda g}\right) d\eta \end{aligned} \quad (14)$$

This result shows that, provided that the pupil function in both variables x_o and y_o is the same, a single 2-D WDF contains the necessary information for obtaining the entire set of elemental images of any arbitrary 3-D object. The intensity distribution in the pick-up plane can be obtained through line integrals of the WDF of the pupil function, where the frequency variable is a linear function of the space variable. Thus, each value of the axial distance z defines the slope of the straight line in the phase space $(z/\lambda d(d+z))$, whereas the image point coordinate $(\mathbf{x}')$ determines the frequency (ν) intersects: $\mathbf{x}'/\lambda g$ of the line.

From theoretical point of view (14) represents a very efficient method for the numerical computation of the elemental images, since the whole intensity distribution in the image plane provided by a 3-D object can be obtained from a single 2-D phase space distribution, even for different wavelengths (note that the wavelength is a parameter in (14)). For the numerical computation of the WDF on arbitrary line segments several fast computational methods have been recently proposed (see, for example, [29]).

On the other hand, the feature that a single 2-D WDF associated to the pupil function contains information of the system response to DOF can be used to perform a visual analysis of the II system performance. In fact, to have an optical imaging system with extended DOF (or that is insensitive to changes of focus), the contribution to the image plane of object points at different distances z should be nearly constant. Since in our approach, each value of z defines the slope of the integration lines in (14), to get nearly constant values for the intensity from a given WDF, it must be slowly varying function with z over a relatively wide angular region about the x_0 axis. Thus, by simply looking at the WDF chart one is able to judge the tolerance of the system to different depths of field (this fact is illustrated in the next section). A very important feature of the analysis of 3-D II performed using the WDF representation is its graphical interpretation. This property allows us, to use the same reasoning used before in the analysis of the performance of the system, now for designing pupil functions that increase the DOF (see Section IV). Note that the formalism presented here can be applied not only for the analysis and synthesis of amplitude pupils but is of general validity to handle also phase pupils [30].

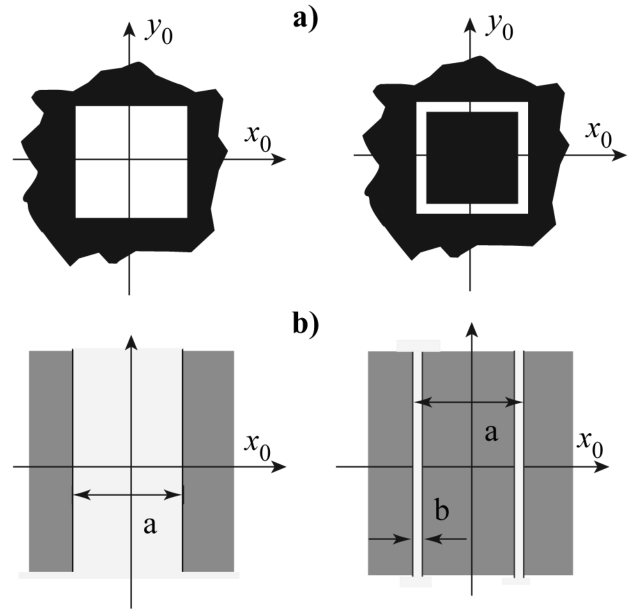

Fig. 2. (a) Square pupil and square pupil with central obscuration. (b) 1-D functions associated to the pupils in (a).

IV. Extension of the Depth of Field in a II System

As an application of the theory developed in the previous section let us compare the performance of a II system with lenslets having square pupils with another having a square pupils with central squared obscurations. The pupil functions of the lenslets of these two systems are described by

$$p(\mathbf{x_o}) = \mathrm{rect}\left(\frac{\mathbf{x_o}}{\mathrm{a}}\right) \quad (15)$$

and

$$q(\mathbf{x_o}) = \mathrm{rect}\left(\frac{\mathbf{x_o} - (\mathrm{a}-\mathrm{b})/2}{\mathrm{b}}\right) + \mathrm{rect}\left(\frac{\mathbf{x_o} + (\mathrm{a}-\mathrm{b})/2}{\mathrm{b}}\right), \quad (16)$$

respectively (see Fig. 2). Using (1), (5), (6) and (7) the WDF of the 1-D version of functions in (15) and (16) result

$$W_{p,a}(x_o; \nu) = \mathrm{rect}\left(\frac{x_o}{\mathrm{a}}\right)\frac{\sin 2\pi\nu\,(\mathrm{a} - 2|x_o|)}{\pi\nu} \quad (17)$$

and

$$\begin{aligned} W_q(x_o; \nu) = &W_{p,b}\,(x_o - (\mathrm{a}-\mathrm{b})/2; \nu) + W_{p,b}\,(x_o + (\mathrm{a}-\mathrm{b})/2; \nu) \\ &+ 2W_{p,b}(x_o; \nu)\cos\left(2\pi\nu(\mathrm{a}-\mathrm{b})\right). \end{aligned} \quad (18)$$

Gray level pictures of these two phase-space distributions are represented in Fig. 3. From these figures we can notice that the pupil with the central obscuration [see Fig. 3(b)] is a slowly

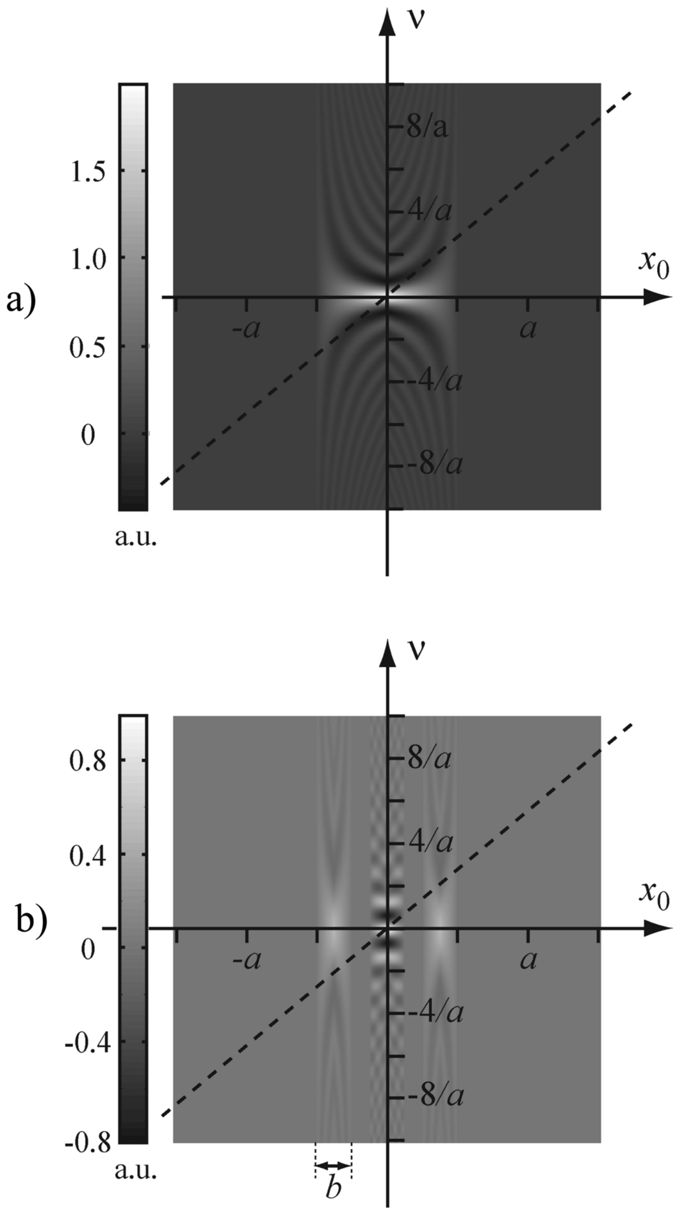

Fig. 3. WDF of the lenslet pupil functions in Fig. 2(b). (a) Square pupil lenslet and (b) obscured square pupil lenslet. The slope of the line in the figures are computed for the same value of the axial distance ($z1 = 20$ mm). Different values of DOF are associated with different values of z (slopes of the lines, in the same WDF.

varying function over a relatively wide angular region about the x_0 axis as compared with the clear function. This means that it produces an extension of DOF of the system. To verify this, let us compare numerically the contribution to the image plane of an object point located at an arbitrary distance z_1 (see Fig. 1) to the contribution of a point at object reference plane located at $z = 0$. The theoretical values of the intensity provided for both systems at z_1 are obtained by adding the values of the WDFs along the straight lines in Fig. 3 (z_1 fixes the slope of the line according to (14)). It is clear that in the range between $z = 0$, and $z = z_1$ (slopes between 0 and $z_l/\lambda d(d + z_l)$) the WDF of the obscured pupil varies slower than the WDF of the clear pupil providing more uniform values of the intensity. Therefore, the quality of the final image is less degraded with misfocus.

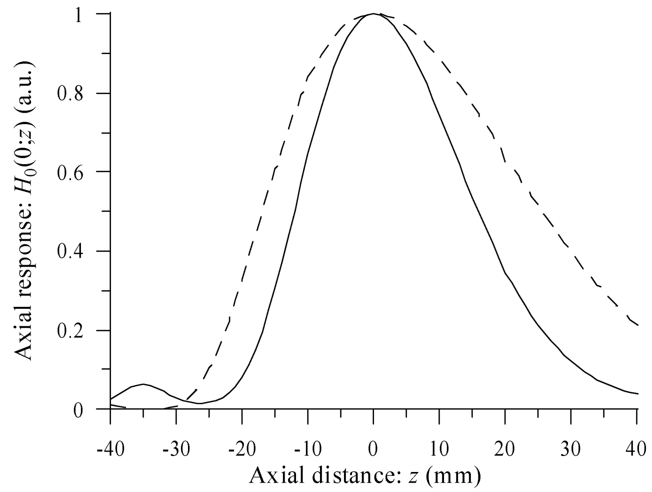

Fig. 4. Values of the irradiance computed at the axial point of the image plane produced by a point object located at different distances z from the reference object plane in Fig. 1 using a clear square pupil (solid line) and with the proposed obscured pupil (dashed line) in this example $b/a = 0.25$.

These assumptions are confirmed by the numerical computation (using (14)) of the intensity at the axial image point for a typical II system using the two pupil functions in Fig. 2. In the computation we used the following parameters $d = 100$ mm, $\lambda = 500$ nm, $a = 1$ mm, and $b = 0.25$ mm (see Figs. 1 and 2). The results are shown in Fig. 4, where the values of the intensity are normalized to unity. The gain in the DOF for this particular example can be visualized by noting that for the extreme value $z = 40$ mm, the relative axial intensity for the obscured pupil is 5 times higher than the one for the clear pupil. This result, that confirms our predictions in Section III, can also be understood by taking into account that the square pupil with the central obscuration can be considered as the separable version of an annular aperture which improve the longitudinal resolution respect to of the clear circular pupil [14], [31]. In our proposal, as the value of b approaches to zero the DOF increases; however, there exist a trade off between the gain in DOF and the loss of both: transversal resolution and light throughput in the image plane. Therefore, the optimum ratio of b/a depends on the particular setup. The maximum DOF we want to achieve limits the highest value of b compatible with it.

V. Conclusions and Future Perspectives

In this paper, the WDF formalism has been applied for the first time to the analysis of 3-D II systems. I was shown that the intensity at the image plane of a general II system can be obtained by means of the WDF of the microlenses pupils. Moreover, for separable pupils, a single WDF, defined in a 2-D phase space, contains the information to compute the intensity for a general 3-D object. The intensity is obtained as line integrals in the phase-space. The slopes of these lines are related to the DOF of the system. Results showed that the WDF is a promising approach because it enables visual discrimination between the effects on the DOF with different pupils of the lenslets. The usefulness of our proposal is demonstrated by proposing a simple separable pupil function that increases the DOF of a conventional II system.

A very important feature of this approach is that it provides physical insights that allow designing pupil masks for extending the DOF. Besides, other aspects of practical interest that has not been studied for II system, such as the optical aberrations of the lenslets, can be tackled with this powerful theoretical tool. Furthermore, there is a 2-D Fourier transform relationship between the WDF and the Woodward ambiguity function [21]. This latter function contains simultaneously all the optical transfer functions (OTFs) associated with an optical system with varying focus errors [23]. Thus, it is possible to use the OTF as a visual merit function of II systems. All these topics will be developed in future works.

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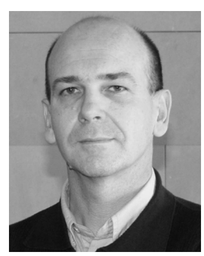

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