Spectral anomalies in focused waves of different Fresnel numbers

Carlos J. Zapata-Rodríguez

Department of Optics, University of Valencia, C/Dr. Moliner, 50. 46100 Burjassot, Spain

Juan A. Monsoriu

Department of Applied Physics, Polytechnical University of Valencia, Camino de Vera S/N. 46022 Valencia, Spain

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Light propagation induces remarkable changes in the spectrum of focused diffracted beams. We show that spectral changes take place in the vicinity of phase singularities in the focal region of spatially coherent, polychromatic spherical waves of different Fresnel numbers. Instead of the Debye formulation, we use the Kirchhoff integral to evaluate the focal field accurately. We find that as a result of a decrease in the Fresnel number, some cylindrical spectral switches are geometrically transformed into conical spectral switches. © 2004 Optical Society of America

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1. INTRODUCTION

Recently a number of papers have focused on an optical effect characteristic of polychromatic waves that has been called spectral switch. First the effect appeared in partially coherent fields,[1,2] but soon it was demonstrated that this is not a correlation-induced but a diffraction-induced effect. Propagation causes changes in the spectrum of diffracted beams, and, in some cases these alterations may be observed clearly in the form of spectral shifts and spectral switches. Wolf and collaborators[3–7] have investigated the spectral switch effect in fully coherent beams. In particular, they considered a polychromatic focused scalar wave that is diffracted by a circular aperture. The spectral distribution suffers significant distortion in the vicinity of phase singularities, where the incident spectral peak is split into two sidelobes. Spectral switches associated with phase singularities are then found along the optical axis and in the transverse focal plane. It is important to note that these results hold for high-Fresnel-number arrangements. In contrast, it was reported that depolarization may cause these spectral switches to disappear in high-numerical-aperture lenses.[8]

High-Fresnel-number focal fields are evaluated by means of the classic Debye theory. In the framework of this formulation, the irradiance distribution presents inversion symmetry about the geometrical focus, where the maximum value is reached. Additionally, the locus of spectral anomalies in polychromatic focused waves also evidences this inversion symmetry.[3] However, low-Fresnel-number focal waves exhibit severe discrepancies.[9–12] For instance, the point of maximum irradiance is not at the geometrical focus but is shifted toward the diffracting aperture. This is the so-called focal shift. Other effects may be present in low-Fresnel-number focusing arrangements such as focal switches[13] and inverse focal shifts.[14] Instead of the Debye formulation of focused fields, we should use the Fresnel–Kirchhoff integral to accurately evaluate the three-dimensional amplitude distribution of the field in the focal region.[15,16] In this paper we demonstrate that spectral switches exist in low-numerical-aperture lenses, but observable asymmetries are found when the Fresnel number of the focusing geometry is close to and lower than unity.

The organization of the paper is as follows. In Section 2 we revise the Fresnel–Kirchhoff formulation to determine the three-dimensional amplitude distribution of focal waves of different Fresnel numbers. In Section 3 we evaluate the diffraction-induced spectral shift in the focal region, and we find a spectral switch effect near phase singularities. It is remarkable that by means of some dimensionless spatial coordinates, the relative spectral shift does not depend on the Fresnel number of the focusing setup. In Section 4 we display the spectral shift spatial distribution in the vicinity of the geometrical focus of different-Fresnel-number fields. We reproduce some previous results for high Fresnel number and find significant deviations when the Fresnel number decreases. We also focus our attention on nearly plane waves diffracted by an opaque screen where the Fresnel number vanishes. This specific case has been investigated previously,[6] and again our results agree exactly. Finally, we summarize the novel results recounted in the paper.

2. DIFFRACTION PATTERNS IN THE FOCAL REGION

First we consider a monochromatic focused wave that is diffracted by a circular aperture. The diffracting screen is located in a plane transverse to the optical axis and centered on it. The aperture radius is denoted by a, and the distance from this diffracting aperture to the geo-

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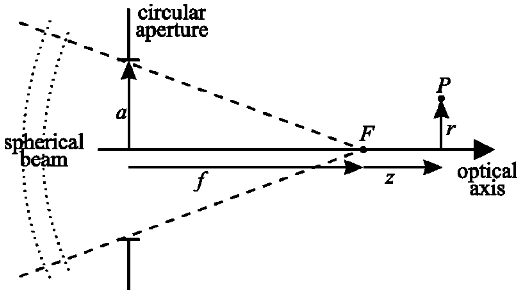

Fig. 1. Schematic diagram of the optical setup.

metrical focus of the spherical beam F is given by the focal distance f, as depicted in Fig. 1. The three-dimensional amplitude distribution for a monochromatic scalar wave in the focal region is evaluated by means of the Fresnel–Kirchhoff diffraction integral written as[16]

$$U(u, v) = B(u)\exp \phi(u, v) \int_0^1 \mathrm{J}_0(v\rho)\exp\left(-i\frac{u}{2}\rho^2\right)\rho \mathrm{d}\rho, \tag{1}$$

where

$$(u, v) = 2\pi N\left(\frac{z/f}{1 + z/f}, \frac{r/a}{1 + z/f}\right) \tag{2}$$

are normalized axial and transverse spatial coordinates, assuming that the geometrical focus is located at cylindrical coordinates $z = 0$ and $r = 0$. The term $\phi(u, v)$ involves a phase factor that is irrelevant in the present study, and

$$B(u) = -\frac{2\pi i}{\lambda}\left(\frac{a}{f}\right)^2\left(1 - \frac{u}{2\pi N}\right)A(\omega) \tag{3}$$

takes into account the attenuation of the Huygens wavelets emerging from all the points of the diffracting screen, where A is the incident wave amplitude. The parameter

$$N = a^2/\lambda f \tag{4}$$

is the so-called Fresnel number of the focusing geometry, where λ is the free-space wavelength of the incident monochromatic radiation; it provides the number of half-wave zones of the aperture as observed from the focus F.[17]

When we consider polychromatic radiation, the squared modulus of the amplitude $A(\omega)$ in Eq. (3) represents the spectral strength of the incident radiation. The three-dimensional amplitude distribution given in Eq. (1) explicitly depends on the frequency of the incident radiation $\omega = 2\pi c/\lambda$, which may be rewritten as $U(u, v; \omega)$ The spatial variables u and v and the Fresnel number N are also frequency dependent. To formulate these frequency dependencies for such variables and parameters we write the following expressions,

$$N = \frac{\omega}{\omega_0}N_0, \tag{5}$$

$$(u, v) = \frac{\omega}{\omega_0}(u_0, v_0), \tag{6}$$

where N_0 is the Fresnel number for a characteristic frequency ω_0. Also, $u_0 = u(\omega_0)$ and $v_0 = v(\omega_0)$. For the following discussion note also that

$$B(u; \omega) = \frac{B(u_0; \omega_0)}{\omega_0 A(\omega_0)}\omega A(\omega); \tag{7}$$

i.e., the term B takes the frequency modulation of the incident spectral amplitude $A(\omega)$ apart from a linear factor. The three-dimensional polychromatic irradiance distribution is determined as the squared modulus of Eq. (1) and may be written as

$$S(u_0, v_0, \omega) = |U(u, v; \omega)|^2 = S^{(i)}(\omega)M(u_0, v_0, \omega), \tag{8}$$

where we have introduced the spectrum of the incident radiation as

$$S^{(i)}(\omega) = \frac{|A(\omega)|^2}{f^2}, \tag{9}$$

and the factor $M(u_0, v_0, \omega)$ is called the spectral modifier, which is responsible for spectral alterations due to diffraction. We assume that the incident focusing beam has a Gaussian spectral distribution of the form

$$S^{(i)}(\omega) = S_0 \exp\left[-\frac{(\omega - \omega_0)^2}{2\sigma^2}\right], \tag{10}$$

where S_0 is a constant. Thus the central frequency of the Gaussian distribution is denoted by ω_0, and its width is given by σ; see caption of Fig. 2. A Lorentzian spectral distribution may also be used, and conclusions equivalent to those of the following analysis would be obtained.

The diffraction integral given in Eq. (1) holds for the paraxial regime, where the numerical aperture of the focusing optical system is low. Also, the Fresnel number usually takes high values, which involves some standard simplifications. In the vicinity of the geometrical focus, the spatial coordinates u and v may be approximated as linear functions of the axial and radial coordinates, respectively:

$$(u, v) = 2\pi N(z/f, r/a). \tag{11}$$

Additionally, the term B reaches constant values in terms of the spatial variable u. The above approximations are taken in the Debye formulation of monochromatic focused

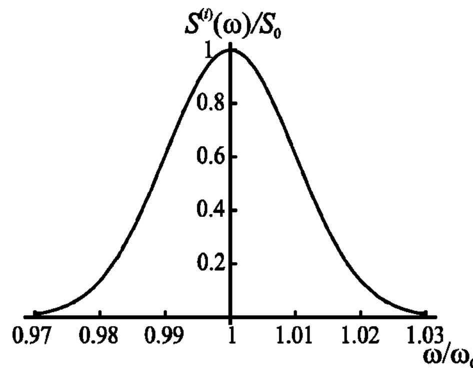

Fig. 2. Gaussian power spectrum of the incident radiation. The central value is given by $\omega_0 = 10^{15}\,\mathrm{s}^{-1}$ and the spectral width is $\sigma = 10^{13}\,\mathrm{s}^{-1}$.

waves.[18] Under these circumstances, the three-dimensional irradiance distribution exhibits some symmetries.[19] The geometrical focus exhibits maximum irradiance value in the focal region, and the irradiance possesses inversion symmetry about the focal point. This is responsible for the fact that a pair of transverse irradiance patterns that are equidistant from the focal plane have the same spatial distribution.[20] Finally, spatial symmetries always appear for any frequency of the incident radiation, assuming that the high-Fresnel-number requirement is fully satisfied. As a consequence, polychromatic focused waves also present the same irradiance symmetries about the geometrical focus.

3. POWER SPECTRUM IN THE VICINITY OF THE GEOMETRICAL FOCUS

Equation (8) shows that propagation induces changes in the spectrum of focused diffracted beams. Particularly remarkable are the spectral shifts and spectral switches found near phase singularities of high-Fresnel-number focused waves. We are interested in extending the study of these spectral anomalies to focal waves of different Fresnel numbers. Then we define the relative spectral shift[3] in the focal region as

$$\frac{\delta\omega}{\omega_0} = \frac{\bar{\omega} - \omega_0}{\omega_0}, \tag{12}$$

where

$$\bar{\omega}(u_0, v_0) = \frac{\int \omega' S(u_0, v_0, \omega')\mathrm{d}\omega'}{\int S(u_0, v_0, \omega')\mathrm{d}\omega'} \tag{13}$$

is the first-order spectral moment, which represents the mean value of the spectral distribution at a specific spatial coordinate (u_0, v_0). It is remarkable that in the $u_0 v_0$ coordinate system the relative spectral shift does not depend on the Fresnel number N_0 of the focusing setup. In this space the only dependence of $S(u_0, v_0, \omega)$ on the Fresnel number comes from the term $B(u; \omega)$ shown in Eq. (3). According to Eq. (7), B may be factorized into two terms: One expresses the frequency dependence of B, which is independent of the Fresnel number N_0, and the other includes the Fresnel number of the focusing setup but is frequency independent. As a consequence, the Fresnel number N_0 is an irrelevant parameter in the evaluation of $\bar{\omega}$ and consequently in the relative spectral shift for a given pair (u_0, v_0). We may conclude that in the $u_0 v_0$ geometry, the spatial distribution of the relative spectral shift is invariant to a change of the Fresnel number of the focal waves in the frame of the Fresnel–Kirchhoff diffraction theory.

The relative spectral shift for the incident power spectrum of Fig. 2 is plotted in Fig. 3. Note that although v_0 is a nonnegative spatial variable in the focal volume [see Eqs. (2) and (11)], we have made use of the axisymmetry property of the optical arrangement, and we should interpret negative values of v_0 as points with the same transverse and axial spatial coordinates, r and z, as their positive counterparts. Dramatic spectral distortions are

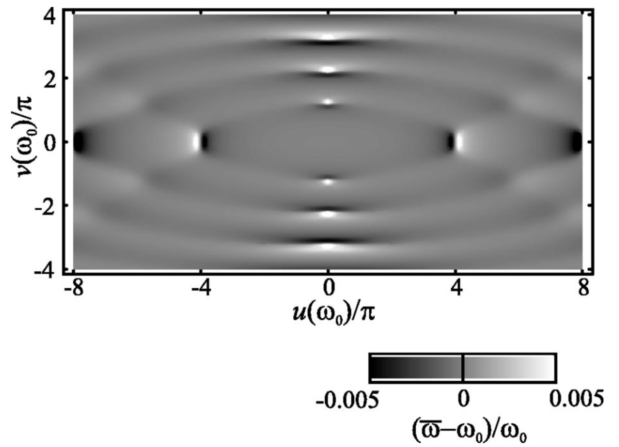

Fig. 3. Gray-coded plot of the relative spectral shift $\delta\omega/\omega_0$ in the focal region as a function of normalized spatial variables $u(\omega_0)$ and $v(\omega_0)$. The spectral Gaussian parameters are given in Fig. 2.

observed in the vicinity of phase singularities. Spectral switches associated with these phase singularities are found along the optical axis and in the transverse focal plane. For example, we find a phase singularity for ω_0, which is the central value of the Gaussian power spectrum of the incident radiation, in the first zero of the Airy pattern given by $v_0 = 1.220\pi$. In Fig. 4(a) it is shown that the spectrum distribution at this point is split into two sidelobes. At neighboring points on the transverse focal plane the spectrum distribution suffers pronounced spectral shifts. The relative spectral shift takes a minimum value of $(\delta\omega/\omega_0)_{\min} = -0.00996$ at $v_0^{\min} = 1.220\pi - \Delta$ [see Fig. 4(b)] and a maximum value of $(\delta\omega/\omega_0)_{\max} = +0.00978$ at $v_0^{\max} = 1.220\pi + \Delta$ [see Fig. 4(c)], where $\Delta = 0.012\pi$. The overall relative spectral shift in the neighborhood of this phase singularity gives

$$\left(\frac{\delta\omega}{\omega_0}\right)_{\max} - \left(\frac{\delta\omega}{\omega_0}\right)_{\min} = 0.0197, \tag{14}$$

which involves a drastic spectral change of nearly 2% in a region bounded by a thin ring of width $v_0^{\max} - v_0^{\min} = 2\Delta$.

The power spectrum has a maximum at a frequency that switches at points with phase singularities, which produces a spectral discontinuity. However, we have defined the relative spectral shift $\delta\omega/\omega_0$ given in Eq. (12) as a continuous function. In these terms, only rapid spectral changes are detectable in the neighborhood of phase singularities. These singularities are located on the transverse focal plane and along the optical axis, where $\delta\omega = 0$. Figure 5(a) shows the relative spectral shift in the transverse focal plane in terms of the spatial coordinate $v(\omega_0)$. Spectral switches are observed at points given by $v(\omega_0) \approx (n + 1/4)\pi$, and n is a nonzero positive integer where phase singularities for ω_0 are located. Figure 5(b) illustrates the relative spectral shift along the optical axis in terms of the spatial coordinate $u(\omega_0)$. In this case, rapid transitions of spectral shifts are observed at $u(\omega_0) = 4\pi m$, where m is a nonzero integer where we find phase singularities for ω_0. In both cases, inversion symmetries of spectral shifts about the geometrical focus, $u_0 = v_0 = 0$, are notable in the spatial representation

u_0v_0. Also, note that phase singularities are not the only points with zero spectral shift. However, only two-lobe spectra are able to produce a true spectral switch. This requirement also holds for Fig. 3 at points along vertical lines crossing axial singularities and horizontal lines crossing focal plane singularities.

Finally, we focus our attention on the transverse focal plane. It is well known that the amplitude distribution in the focal plane of a monochromatic focused wave corresponds to an Airy disk pattern. Phase singularities form circular rings, and their radii depend on the incident radiation frequency. Polychromatic radiation shows that the achromatic position of the geometrical focus F involves detuned Airy rings. This is responsible for the great spectral changes observed in the focal plane.[3] Moreover, the Fraunhofer pattern of a polychromatic plane wave diffracted by a circular aperture presents the same Airy pattern, and equivalent spectral shifts and spectral switches near phase singularities are observed.[6] This agrees with the fact that a diffracted plane wave

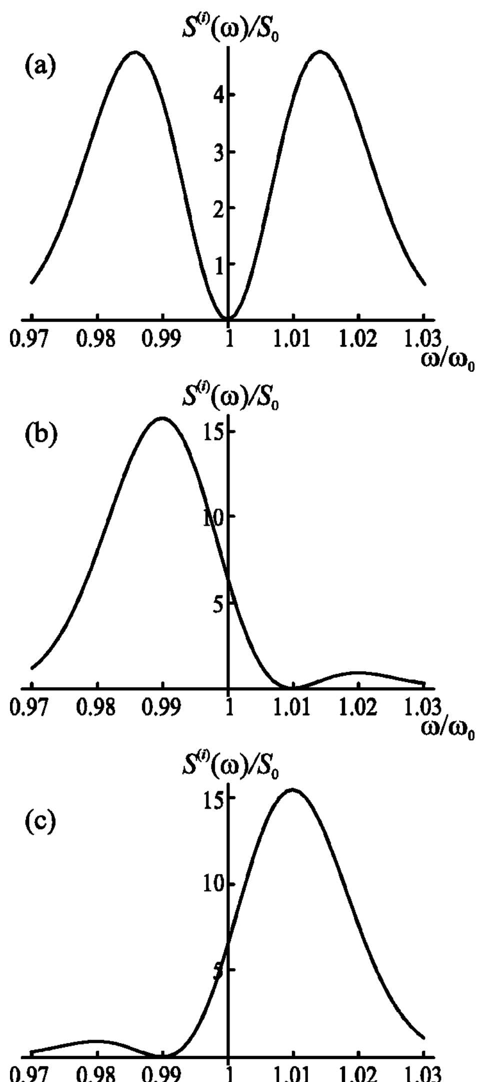

Fig. 4. Normalized power spectrum at transverse points $u_0 = 0$ and (a) $v_0 = 1.220\pi$, (b) $v_0 = 1.220\pi - \Delta$, and (c) $v_0 = 1.220\pi + \Delta$, where $\Delta = 0.012\pi$.

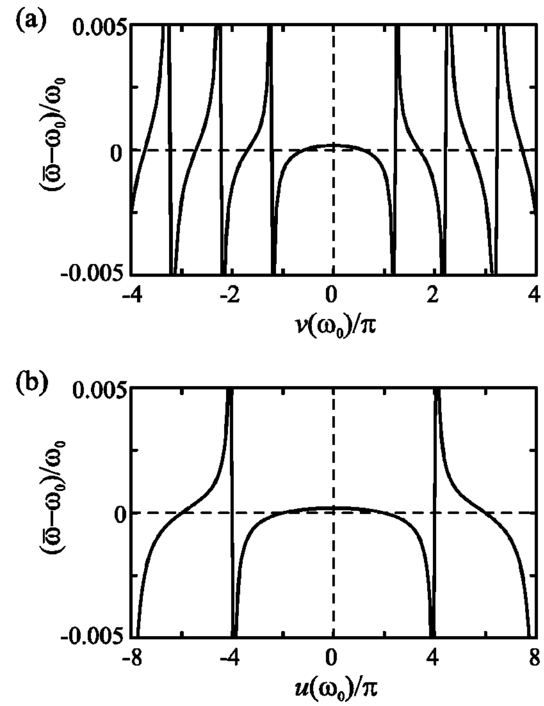

Fig. 5. Relative spectral shift (a) in the transverse focal plane and (b) along the optical axis.

may be viewed as a zero-Fresnel-number focused wave, and, as concluded previously, the same spectral changes should be observed for any value of the Fresnel number. In other words, the Fresnel number may not cause the creation and annihilation of the remarked anomalies in the spectra of in-focus and out-of-focus transverse planes.

4. SPATIAL ASYMMETRY OF SPECTRAL SHIFTS AND SPECTRAL SWITCHES

Figure 3 shows that the relative spectral shift presents symmetries about the geometrical focus F in the u_0v_0 coordinate system. However, we should take into account that these normalized spatial coordinates given in Eq. (2) do not depend linearly on the cylindrical spatial coordinates r and z. Accordingly, there exists a geometrical mapping of the obtained three-dimensional distribution of the relative spectral shift. This kind of spatial transformation has been previously reported[20] for the focal field amplitude. Phase singularities in the focal region are again located at certain points of the focal plane and along the optical axis. Moreover, axisymmetry of the optical setup guarantees that transverse in-focus singularities hold the inversion symmetry about the geometrical focus F. Finally it is expected that the locus where diffraction-induced spectral anomalies are observed suffers lack of symmetry in terms of the axial variable z.

In Fig. 6 we plot the relative spectral shift $\delta\omega/\omega_0$ by using the spatial coordinates z/f and r/a. Again, negative values of r/a are attributed to radially symmetrical points about the optical axis. Axial symmetries about the geometrical focus F are observable in Fig. 6(a), where $N_0 = 100$ takes a high value. This plot should be compared with Fig. 3; the two representations give equivalent results since in this case the Debye formulation holds.

Then Eq. (11) provides the scaling of the respective spatial coordinates employed. This leads us to the relevant conclusion that equivalent results are obtained in the evaluation of the spectral shifts and spectral switches shown in particular normalized spaces by using the paraxial Debye formulation[3] and the more general Fresnel–Kirchhoff diffraction theory. However, this assessment cannot be generalized to the evaluation of the three-dimensional irradiance distribution in the focal region.[20]

In Fig. 6(a) the locus of points with spectral switch approximately describes straight lines crossing phase singularities for ω_0. For axial phase singularities these spectral lines are parallel to the r/a axis. Axisymmetry of the optical arrangement implies that these points are in fact located at transverse planes. This situation may be called plane spectral switches. Moreover, all these points are encircled by disks, and the closest-to-focus one has a radius $r/a = 9 \times 10^{-4}$. It may be demonstrated that $r/a \approx 0.09/N_0$ for a high value of the Fresnel number, $N_0 \gg 1$. For phase singularities in the transverse focal plane the spectral switches are generated as lines parallel to the axis. Again, axisymmetry of the optical system implies that these points are located at cylindrical surfaces whose radii are determined by the phase singularities for ω_0 in the focal plane. This case may be named cylindrical spectral switches.

The relative spectral shift for intermediate values of the Fresnel number that are close to unity is plotted in Figs. 6(b)–6(e). Again, abrupt changes in the spectral shift are found. We observe that plane spectral switches are still discernible. However, a decrement in area of the

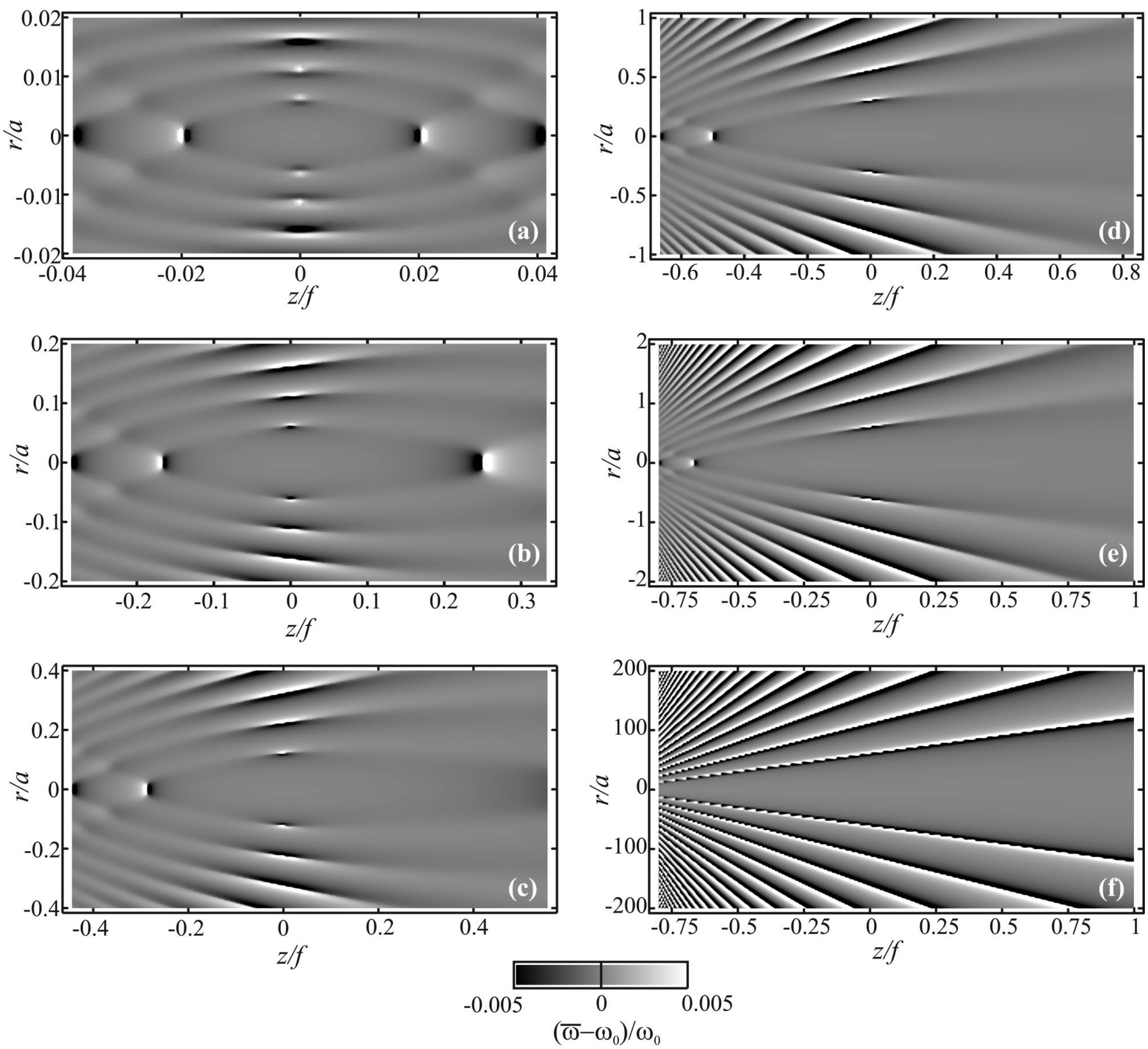

Fig. 6. Gray-coded plots of the relative spectral shift in the focal region for (a) a high Fresnel number $N_0 = 100$, some intermediate Fresnel numbers: (b) $N_0 = 10$, (c) $N_0 = 5$, (d) $N_0 = 2$, (e) $N_0 = 1$, and (f) a nearly flat beam characterized by a Fresnel number $N_0 = 0.01$.

circular region where these spectral switches are located is notable for axial phase singularities placed between the diffracting aperture and the geometrical focus. Moreover, zero spectral shifts are not observed at points along horizontal lines crossing focal plane singularities. A conical locus for zero spectral shifts establishes the spatial obliquity of the spectral switches. Then cylindrical spectral switches are geometrically transformed into conical spectral switches. The apices of all these conical surfaces are situated at the same point of the optical axis, given by $z/f = -1$. In fact, this point belongs to the transverse plane where the diffracting screen is placed.

Figure 6(f) displays the relative spectral shift for $N_0 = 1/100$. This situation corresponds to nearly plane waves diffracted by an opaque screen. Perfect plane waves would be characterized by a zero Fresnel number. Thus the transverse focal plane may be viewed as the Fraunhofer pattern of the diffracting aperture. Therefore conical spectral switches correspond to the far-field off-axis phase singularities of the diffracted pattern, i.e., phase singularities of the Airy pattern for ω_0. This case has been investigated elsewhere,[6] and these conical spectral switches were found.

5. SUMMARY

We have considered a polychromatic focused wave that is diffracted by a circular aperture and found that for any value of the Fresnel number, the spectrum distribution suffers dramatic distortions in the vicinity of phase singularities, where the spectrum is split into two sidelobes. First we demonstrated that with a normalized $u_0 v_0$ coordinate system the relative spectral shift does not depend on the Fresnel number of the focusing setup. However, we should take into account that the normalized spatial coordinates implicitly depend on the Fresnel number. As a consequence, axial phase singularities in the focal region are relocated at certain points along the optical axis, and the locus of points where diffraction-induced spectral anomalies are observed suffers lack of symmetry about the focus.

For high Fresnel numbers, the loci of points with spectral switches are classified into two categories. The first is called plane spectral switches since some of these points are located at transverse planes that cross axial phase singularities. In the second, the points are located at cylindrical surfaces whose radii are determined by the phase singularities in the transverse focal plane. This class is named cylindrical spectral switches. When the Fresnel number is decreasing, plane spectral switches are still observable. However, cylindrical spectral switches are geometrically transformed into conical spectral switches, whose apices are situated at the axial point of the diffracting screen. Finally, we focused our attention on plane waves diffracted by an opaque screen where the Fresnel number vanishes. Thus the transverse focal plane may be viewed as the Fraunhofer pattern of the diffracting aperture, and therefore conical spectral switches are associated with far-field off-axis phase singularities.

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Carlos Zapata-Rodríguez can be reached by e-mail at carlos.zapata@uv.es.

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