PHOTONIC STRUCTURES

Fractal Zone Plates Produce Axial Irradiance With Fractal Profile

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Certain natural phenomena exhibit distinctive features that can be associated with the concept of fractal, and their study has become a matter of great interest for scientists in many fields. Recent achievements in optics range from the observation of the fractal structure in the transverse modes of certain unstable lasers[1] to the observation of the self-similar structure of the transverse diffraction patterns generated from fractal[2] and even non-fractal apertures.[3] We have recently reported a new type of diffracting 2D objects with radial symmetry that show multiple foci with internal fractal properties along the optical axis. Since these objects with internal fractal structure can be constructed from conventional zone plates, we call them fractal zone plates.[4]

As is well known, conventional Fresnel zone plates consist of alternate transparent and opaque concentric rings, the radii of which are proportional to the square root of natural numbers. Thus, one way to realize the construction of the transmittance of a Fresnel zone plate is by using a one-dimensional (1D) binary periodic function $q(\zeta)$ like one of those shown in Fig. 1(a). If this function is represented using a new variable $r_0 = \sqrt{\zeta}$ and the whole structure is rotated around one extreme, the resulting object is a Fresnel zone plate [see Fig. 1(c)]. In a similar way, if the generating function $q(\zeta)$ has a fractal profile rather than a periodic profile (see, for instance, any element of the regular Cantor set in Fig. 1(b), the resulting radially symmetric function is a fractal zone plate (FZP) [see Fig. 1(d)]. Thus, this new type of zone plate has fractal structure along the square of the radial coordinate. Note that each periodic function in Fig. 1(a) has an associated regular fractal structure in Fig. 1(b), so that a FZP can be understood as a Fresnel zone plate with some missing clear zones [compare Fig.1(d) with Fig. 1(c)].[4]

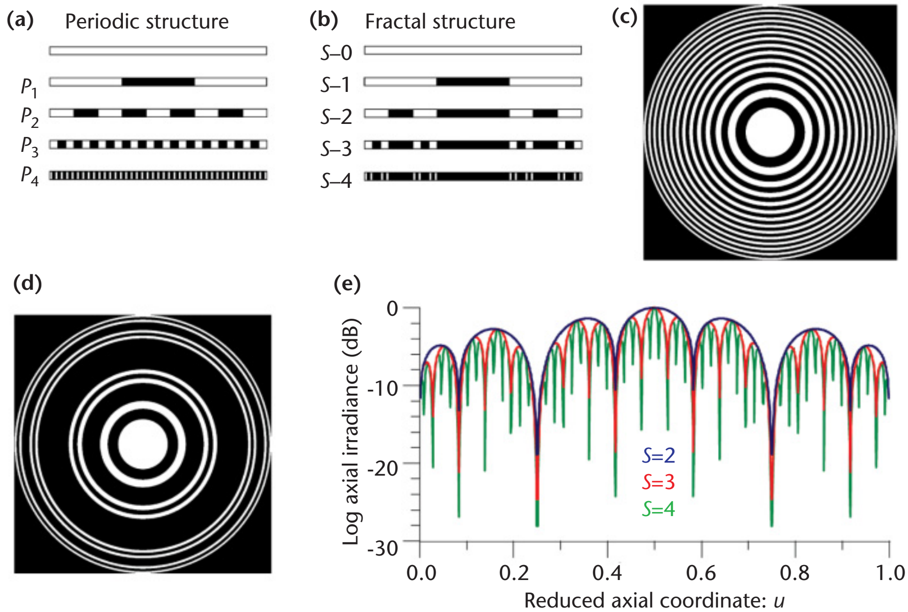

Figure 1. Schemes for the generation of the binary function $q(\zeta)$ for: (a) Fresnel zone plate with different periods; (b) the associated Cantor FZP. In this representation, clear and dark segments correspond, respectively, to the values 1 and 0 of the generating binary function. The Fresnel zone plate in (c) and the associated FZP in (d) are generated from the 1D functions in (a) and (b) for S=3 by rotating the respective whole structure around one extreme after the change of variables $r_0=\sqrt{\zeta}$. In (e), we show a log plot of the axial irradiances versus the reduced axial coordinate

$$u = \frac{1}{3^S}\frac{a^2}{2\lambda R}$$

where a is the maximun extent of the pupil and R is the axial distance from the pupil plane.

The axial irradiance provided by a given FZP presents multiple foci the main lobes of which coincide with those of the associated Fresnel zone plate. However, the internal structure of each focus exhibits its characteristic fractal profile, reproducing the self-similarity of the originating FZP. In fact, as can be seen in Fig. 1(e) for different FZPs of the same family, the axial irradiance for a given stage of growth S is a modulated version of that associated with the previous stage. As S increases, one encounters an increasing number of zeros and maxima which are scale invariant over discrete dilations (in our case of factor 3) as corresponds to a self-similar structure.

Aside from their theoretical interest, FZPs can find applications in scientific areas such as THz tomography[5] and X-rays,[6] where zone plates are becoming the key elements used to obtain images. Given the simple theoretical relation between the transmittance of FZPs and their axial response,[4] with this kind of zone plate, the synthesis of axial irradiances with fractal profile is now an easy task. The influence of optical aberrations and some construction parameters (such as the lacunarity) on their axial response are subjects of continuing study.

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