

# Optical aberrations measurement with a low cost optometric instrument

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A simple experimental method for measuring optical aberrations of a single lens is proposed. The technique is based on the use of an optometric instrument employed for the assessment of the refractive state of the eye: the retinoscope. Experimental results for spherical aberration and astigmatism are obtained. © 2002 American Association of Physics Teachers.  
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## I. INTRODUCTION

The study of optical aberrations is an important part of undergraduate and graduate optics courses in astronomy, physics, optometry, and optical engineering.<sup>1–4</sup> However, instructors frequently find it difficult to motivate students in its study. Our experience indicates that some introductory comments on the practical importance of aberrations can be very helpful to this end. Let us consider, for example, the following two examples of spherical aberration that are well known to the public. One example is the Hubble Space Telescope launched into orbit in 1990.<sup>5</sup> Scientists soon discovered that the telescope produced severely blurred images having the characteristics of spherical aberration. Three years (and a lot of money) were necessary to repair the telescope, which is still giving the best images of the universe ever seen. Another example where spherical aberration is crucial arises in ophthalmology,<sup>6</sup> where advanced surgical procedures for reducing refractive errors are growing in popularity. With the development of sophisticated computer-controlled laser scanning and delivering systems, surgical techniques are becoming more and more precise. However, if aberrations are not taken into account and refractive surgery is performed based on paraxial assumptions, the cornea takes an aspheric shape, which causes a dramatic increase (up to ten times its natural level) in the spherical aberration of the eye. The consequence is that visual performance, especially in a darkened environment, is highly affected.

Cases like the ones mentioned, followed by the theory, can be complemented by some simple experiments on aberration.<sup>7</sup> These experiments, which are the natural way to verify both the failure of paraxial optics and the importance of aberrations in optical systems, result in a stimulating experience for the students.

Although spherical and chromatic aberrations have been the subject of several educational papers during the past few years,<sup>8–11</sup> other kinds of aberrations, such as astigmatism, have not been considered. Furthermore, to our knowledge there is no single optical arrangement that can be adapted readily to study several kinds of aberrations in the context of teaching. In this note we describe an experiment that is more versatile than previous setups and is an easily implementable adaptation of the Foucault (or knife edge) test.

The Foucault test<sup>12</sup> was originally developed to assess concave mirrors for astronomical use and is still today one of the most widely used methods for the evaluation of the aberrations of optical systems. The technique is usually performed with a device that is especially designed for each application. Based on the same physical principles as the Foucault test (although independently developed), the retino-

scope is a simple and rather inexpensive optometric instrument mainly used to objectively measure the refractive state of the eye.<sup>13</sup> In the experiments reported here, a retinoscope is used to measure both longitudinal spherical aberration (LSA) and astigmatism of a lens. In Secs. II and III we revisit the fundamentals of the Foucault test and the retinoscopic technique, and show that, apart from minor differences, both methods are based on the same optical principles. In Sec. IV we present the optical setup and the experimental results.

## II. THE FOUCAULT TEST

In the Foucault test a collimated light beam is used to illuminate the optical element to be examined (see Fig. 1). A knife edge that can be displaced both axially and transversely is placed in the focal region, perpendicular to the optical axis. Depending on the position of the knife edge, different parts of the illuminating beam are blocked out. An observational system—the eye, for example—is focused onto the exit pupil of the optical element. The form of the observed shadow pattern depends on several factors, namely the distance between the focus and the knife edge, the transverse position of the knife edge, and the aberrations of the optical element.<sup>12</sup> For an aberration-free optical element, the shadow pattern consists of a dark and a bright region separated by a straight edge parallel to the knife edge. If the knife edge is located behind the focus [see Fig. 1(a)] and is transversely displaced, the observed dark region moves in the opposite direction of the knife edge. On the contrary, when the knife edge is placed in front of the focus, the dark region moves in the same direction as the knife edge [see Fig. 1(b)]. Finally, if the knife edge is located at the focal plane of the system and is transversely displaced, the observed pattern changes suddenly from bright to dark without any apparent motion [see Fig. 1(c)]. Thus, the location of the focus can be determined accurately by observation of the shadow patterns.

To make the observed pattern as bright as possible, the illumination beam is formed in practice using an illumination slit, parallel to the knife edge, instead of a point source.<sup>12</sup>

For an aberrated lens, the shadow-pattern structures are no longer divided by a straight edge. Consequently, it is no longer possible to see a completely bright or dark pattern at a given axial position of the knife edge. In addition, each kind of aberration produces a different pattern (see Fig. 2).

## III. PRINCIPLES OF RETINOSCOPY

The retinoscope is a simple self-luminous hand-held instrument used by optometrists in standard clinical procedures

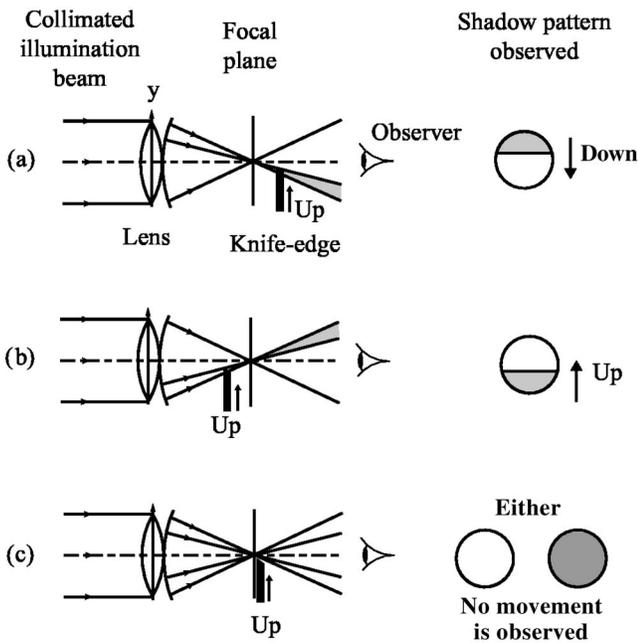


Fig. 1. Knife edge testing of an aberration-free lens. (a) Outside the focus, the knife and the observed shadow move in opposite directions; (b) Inside the focus, the knife and the observed shadow both move in the same direction. (c) On focus no movement of the shadow is observed.

to measure the refractive state of the eye.<sup>13–15</sup> A standard, commercially available streak retinoscope has a light source ( $S$ ), consisting of a straight-filament bulb that is located in the handle of the instrument (see Fig. 3). The light from the source is reflected in a beam splitter toward the patient's eye. Then, a real, and, in general, defocused image of the glowing filament is formed at the retina. This stripe-like spot of light acts as a secondary source of light, playing the role of the illumination slit in the Foucault knife edge. The observer (the retinoscopist) views through the retinoscope pupil ( $P$ ) light that is diffusely reflected by the retina and emerges from the pupil of the eye. In retinoscopy the observed pattern at the patient's pupil is called "the reflex." Tilting the retinoscope (for example, about an axis perpendicular to the plane of Fig. 3) causes the spot to shift its eccentricity from the pupil of the retinoscope and, depending on the refractive state of the eye, to change the appearance of the reflex, or equivalently, the appearance of the shadow pattern that surrounds it. Similar to the Foucault test, the form of the observed pattern depends on the location of the focus of the eye, that is, on the location of its *far point*  $O_R$  in the object space relative to the axial position of the retinoscope.<sup>16</sup> In spite of not being a

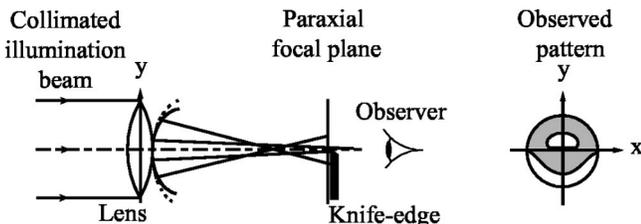


Fig. 2. Knife edge testing of a lens with spherical aberration.

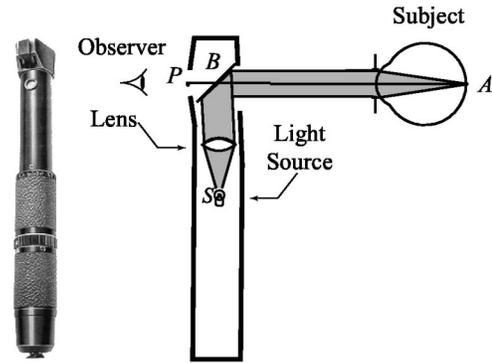


Fig. 3. Photograph of a retinoscope and schematic layout of the elements that compose it. Light coming from the source  $S$  reaches the retina of the examined eye.

straight edge, the edge of the retinoscope pupil acts as the knife edge, because it blocks out the light coming from the secondary source at the retina.

Let us consider the particular case of a myopic eye having its far point  $O_R$  in front of the retinoscope [see Fig. 4(a)]. In this case, a reflex pattern is observed through the pupil of the retinoscope consisting of a central bright fringe surrounded by dark regions at the eye pupil. When the instrument is tilted up, the spot  $A$  also moves up, and its back image  $O_R$  moves down. Hence, the reflex is seen as moving "against" the movement of the instrument [see Fig. 4(b)]. This situation is equivalent to the one represented in Fig. 1(a). On the other hand, if the far point is behind the retinoscope [see Fig. 4(c)], when the retinoscope is tilted up, the observed reflex moves in the same direction. In this case, the movement of the reflex is called "direct." The analogous situation for the Foucault test is the one represented in Fig. 1(b). Finally, as

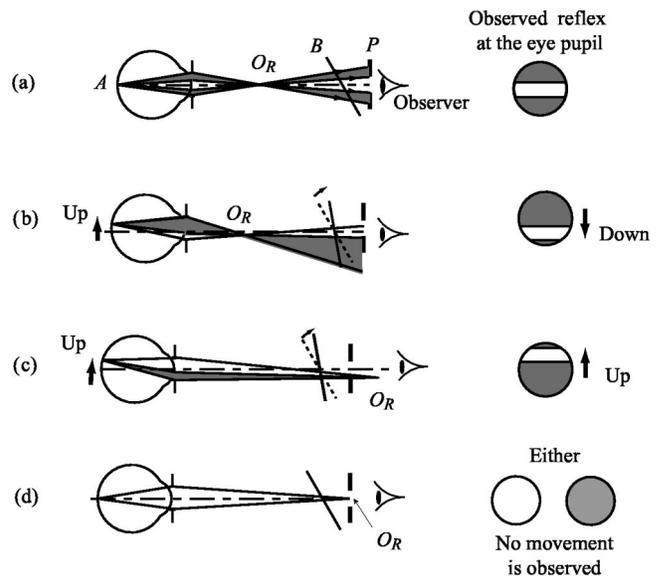


Fig. 4. (a) The light reflected by the retina of a myopic eye with its far point  $O_R$  between the eye and the retinoscope is seen by the observer—the illuminating beam is not shown. (b) When the retinoscope is tilted up, the observed reflex at the eye pupil moves in the opposite direction. (c) If  $O_R$  is located behind the retinoscope and the retinoscope is tilted up, the fundus reflex moves in the same direction. (d) For neutralization  $O_R$  lies just at the plane of the retinoscope pupil. No apparent movement is observed when the retinoscope is tilted.

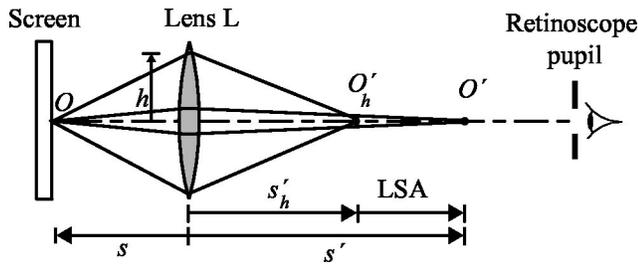


Fig. 5. Scheme of the experimental setup designed to measure the longitudinal spherical aberration of a lens.

happens with the knife edge in Fig. 1(c), when  $O_R$  coincides with the retinoscope pupil and the retinoscope is tilted, the observed pattern suddenly switches from bright to dark without any apparent motion [see Fig. 4(d)]. In optometry, this situation is known as the “neutralization” of the reflex movement, and is the basis of the retinoscopic technique. Thus neutralization is obtained when the object plane (the retina of the eye) is conjugate to the retinoscope pupil plane. This situation can be achieved in practice either by placing trial lenses in front of the eye, or by moving the retinoscope back and forth until the proper distance to the eye is attained. The values of the focal length of the trial lens and the eye–retinoscope distance are sufficient to determine the refractive state of the eye.<sup>13</sup>

The retinoscope is easily operated. In fact, the technique simply consists of the observation of the reflex through the retinoscope pupil while swinging the instrument until neutralization is obtained, that is, until the point is reached where no apparent movement of the reflex is seen.

In summary, the neutralization obtained with a retinoscope is equivalent to the location of the focus of an optical element with the Foucault knife test. Probably because retinoscopy and the Foucault test do not share a common origin, it is not widely recognized that both techniques are in fact based on the same physical principle. Thus, the retinoscope can be used for testing the aberrations of conventional optical elements, as we will show next.

#### IV. ABERRATION MEASUREMENT: BASIC THEORY AND EXPERIMENTAL RESULTS

##### A. Longitudinal spherical aberration

Consider the schematic layout in Fig. 5 in which rays proceeding from the axial point of the screen  $O$  impinge on a spherically aberrated lens at different heights  $h$ . The intersection of the refracted rays with the optical axis depends explicitly on the value of  $h$ . For a given pupil radius, the longitudinal spherical aberration (LSA) is defined as the axial distance between the paraxial image,  $O'$ , and the image given by the outmost rays  $O'_h$ , that is,  $\text{LSA} = s' - s'_h$ . (see Fig. 5). The LSA is related to the geometrical parameters of the lens through the following equation (see, for example, Refs. 1, 2, or 8):

$$L_S = \frac{1}{s'_h} - \frac{1}{s'} = \frac{h^2}{8f^3} \frac{1}{n(n-1)} \left[ \frac{n+2}{n-1} q^2 + 4(n+1)pq + (3n+2)(n-1)p^2 + \frac{n^3}{n-1} \right], \quad (1)$$

Table I. Experimental values of the LSA for  $s = -2f$ .

$h$ (cm)	$\text{LSA}_{\text{exp}}$ (cm) $s = -2f$ , $p = 0$	
	$q = +1$	$q = -1$
1.2	0.8	0.9
2.4	2.6	2.8
3.6	6.8	6.6
4.8	11.6	11.8

where  $n$  is the refractive index of the lens material,  $f$  is the focal length of the lens, and  $q$  and  $p$  are, respectively, the Coddington shape and position factors, defined by

$$q = \frac{r_2 + r_1}{r_2 - r_1}, \quad p = \frac{s' + s}{s' - s} = 1 - \frac{2f}{s'}. \quad (2)$$

In Eq. (2),  $r_1$  and  $r_2$  are the radii of curvature of the two surfaces of the lens, which by convention are both negative when the centers of curvature lie on the source side of the lens.

If we take into account Eq. (2), the LSA can be expressed using certain approximations as:

$$\text{LSA} = s' - s'_h = s' - \frac{s'}{1 + s'L_S} = \frac{s'^2 L_S}{1 + s'L_S} \approx s'^2 L_S. \quad (3)$$

Equations (1) and (3) show that the LSA is proportional to  $h^2$ . The proportionality factor depends on the parameters  $q$  and  $p$ , that is, spherical aberration is governed by the shape and object position factors.

Based on the setup shown in Fig. 5, the goal of the first experiment is to obtain the experimental values of  $s'$  and  $s'_h$  that fit Eq. (3) for different values of  $h$ . To this end we measured the LSA of a plano-convex lens ( $r_1 = \infty$  and  $r_2 = 104$  mm) with focal length  $f = 200$  mm,  $n = 1.48$ , and diameter  $\phi = 115$  mm. To evaluate the paraxial image  $O'$ , a small circular aperture of about 6 mm was first centered at the optical axis just behind the lens L. The distance  $s'$  was determined by displacing a streak retinoscope<sup>15</sup> along the optical axis until reflex neutralization is obtained.<sup>17</sup> To obtain the nonparaxial images  $O'_h$ , the small aperture was transversely displaced and centered at different heights,  $h$ . Thus, for each one of these values, the distance  $s'_h$  was again found by axial displacements of the retinoscope. For the object position factor we choose  $s = -2f$ , that is,  $p = 0$ , for which the theoretical values of LSA for both orientations of the lens  $q = +1$  and  $q = -1$  are the same. Table I summarizes the experimental values of the LSA and these values are plotted versus  $h^2$  in Fig. 6. The linear behavior predicted by Eq. (3) can be clearly seen. In addition, the experimental values for both orientations of the lens are almost the same.

##### B. Astigmatism

To show the versatility of our setup, we also used it to study astigmatism. For this purpose the same lens L was placed on a graded rotation stage to allow the light to impinge obliquely on it. The measurement of the astigmatism is simply the measurement of the axial distance between the tangential ( $O'_T$ ) and sagittal ( $O'_S$ ) images. As can be seen in

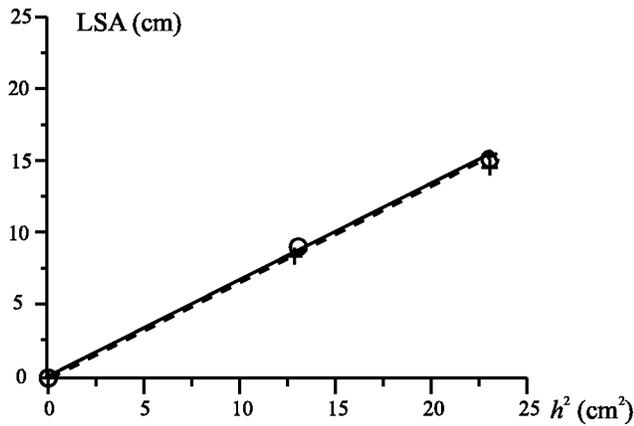


Fig. 6. Representation of the longitudinal spherical aberration (LSA) vs  $h^2$  for  $p=0$ . The circles and the crosses correspond to the experimental values for  $q=+1$  and  $q=-1$ , respectively.

Fig. 7, if the chief ray forms an angle  $\theta$  with the lens axis, the tangential image is perpendicular to the plane of the figure and the sagittal image lies in the plane.

The theory for a thin lens in air predicts that the positions of these images are given by:<sup>1</sup>

$$-\frac{1}{s} + \frac{1}{s'_T} = \frac{1}{\cos \theta} \left( \frac{n \cos \theta'}{\cos \theta} - 1 \right) \left( \frac{1}{r_1} - \frac{1}{r_2} \right),$$

$$-\frac{1}{s} + \frac{1}{s'_S} = \cos \theta \left( \frac{n \cos \theta'}{\cos \theta} - 1 \right) \left( \frac{1}{r_1} - \frac{1}{r_2} \right),$$
(4)

where  $\theta$  and  $\theta'$  are, respectively, the incident and refracted angles in the first lens surface,  $s$  is the object distance, and  $s'_T$  and  $s'_S$  are the tangential and sagittal image positions, respectively. For our purposes it is desirable to use an expression that does not depend on  $\theta'$ . This can be achieved by performing the quotient of Eqs. (3) and (4), which results in

$$\cos^2 \theta = \frac{(s - s'_S)s'_T}{(s - s'_T)s'_S}.$$
(5)

Equation (5) shows that the relative positions of the tangential and sagittal images do not depend on the refractive index of the lens or on its shape factor  $q$ . Moreover, if the object is at an infinite distance, then Eq. (4) gives the position of the tangential and sagittal image foci, and in this case, Eq. (5) reduces to:

$$f_T = f_S \cos^2 \theta.$$
(6)

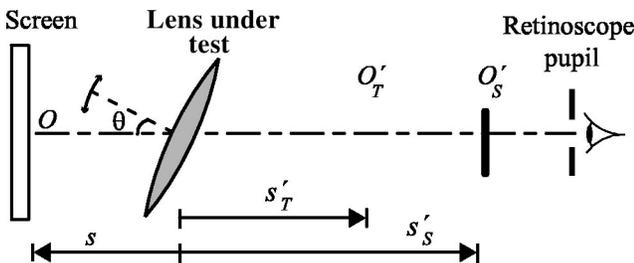


Fig. 7. Schematic layout of the experimental setup to study the astigmatism of a lens (top view). The sagittal image is in the plane of the figure whereas the tangential image is perpendicular to it.

Table II. Influence of the shape factor on astigmatism. The values of  $\theta$  are compared to the ones obtained by introducing the experimental values of  $s'_S$  and  $s'_T$  into Eq. (5).

$\theta$	$\theta_{\text{exp}}$	
	$q=+1$	$q=-1$
15°	16°	16°
25°	26°	24°
35°	34°	36°

If it is desired to check Eq. (6) experimentally, a collimating lens must be added between the screen and the lens under test in the setup of Fig. 7, in order to work with an infinite distance object.

To check the nondependence of astigmatism on the shape factor, we performed the experiment shown schematically in Fig. 7. For the same object distance as in the previous case, the positions of both  $O'_T$  and  $O'_S$  for different values of the incident angle  $\theta$  were found by means of neutralization. In this case, contrary to the LSA, for which the neutralization point was obtained for a single meridian, the neutralization distances needed to measure the astigmatism must be obtained in two perpendicular meridians corresponding to the tangential and sagittal images. To measure the distance  $s'_T$ , the observer must place the retinoscope with the streak perpendicular to the plane of the figure near the lens L. Then it must be displaced along the optical axis examining the horizontal meridian until the neutralization corresponding to  $O'_T$  is obtained. Next, after rotating the streak 90°, the neutralization in the vertical meridian (corresponding to the image  $O'_S$ ) can be obtained in the same way. Two sets of measurements for  $q = \pm 1$  were done for four different values of  $\theta$ . To preserve the paraxial condition, a small circular aperture pupil of about 6 mm was centered at the optical axis just behind the lens. In Table II we compare the actual values of  $\theta$  with the ones obtained by introducing the experimental values of  $s'_S$  and  $s'_T$  into Eq. (5). A second set of measurements was performed to check the nondependence of Eq. (5) on the refraction index. To this end we employed two lenses,  $L_1$  and  $L_2$ , of identical focal length 200 mm, but different refraction indexes,  $n_1=1.5$  and  $n_2=1.7$ , respectively. Table III summarizes the experimental results.

## V. SUMMARY

A new simple approach for the measurement of lens aberrations is proposed. The key element is a streak retinoscope, which is a commercially available optometric instrument based on the same physical principle as the knife edge Fou-

Table III. Influence of the refractive index on astigmatism. The values of  $\theta$  are compared to the ones obtained by introducing the experimental values of  $s'_S$  and  $s'_T$  into Eq. (5).

$\theta$	$\theta_{\text{exp}}$	
	$L_1 (n_1=1.5)$	$L_2 (n_2=1.7)$
20°	20°	19°
30°	31°	29°
40°	41°	40°

cault test. In spite of the fact that the retinoscope is not found in many university optics laboratories, our experience indicates that undergraduate students can use it after a few minutes of instruction. With the proposed experimental technique, we described the measurement of the LSA and the astigmatism of a lens. It was shown that the experimental results agree well with theoretical predictions.

Finally it should be mentioned that this technique could also be extended to the measurement of the longitudinal chromatic aberration of a lens. To this end it is sufficient to insert color filters in the illumination beam and to use an achromatic lens as a collimator. Alternately, inserting blue and red filters allows us to obtain the corresponding focus by neutralization of the retinoscopic reflex.

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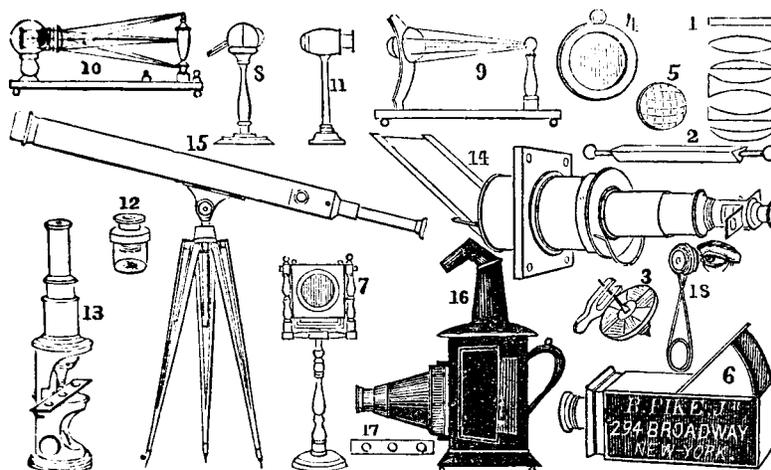
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<sup>15</sup>A description of commercial instruments can be found for example at <http://www.heine.com>.

<sup>16</sup>The far point of accommodation is defined as the point object that is conjugate with the retina when accommodation is fully relaxed. Accommodation may be defined as the adjustment of the optical system of the eye for vision at various distances. See, for example, Ref. 14.

<sup>17</sup>Because the retinoscope uses a white light illumination beam, the experimental values are affected by a small error due to the wavelength dependence of the refraction index. Nevertheless, the neutralization of the reflex can be clearly observed.

## SET OF APPARATUS FOR OPTICS. PRICE \$163.



Set of Apparatus For Optics. The most expensive piece of apparatus is the astronomical telescope (15) with a 3 inch diameter,  $f/20$  objective (\$48). The solar microscope (14) was placed in a blacked-out window; its mirror rotated and tilted to allow sunlight to act as a source for projecting microscope slides on a screen (\$28). The camera obscura (6) was \$4, while the simple microscope (13) was only a dollar. The demonstration eye (11) was used to demonstrate myopia and hypermetropia. The magic lantern (16) could be used with scientific, moral or comic slides. The cost of the set was about 10% of the yearly salary of a college faculty member. (From the 1856 Apparatus Catalogue of Benjamin Pike, Jr. of New York; notes by Thomas B. Greenslade, Jr., Kenyon College)