All-diffractive achromatic Fourier-transform setup

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An achromatic Fourier transformation under broadband converging spherical-wave illumination is optically achieved by use of only two on-axis blazed zone plates. The novel optical configuration provides the achromatic Fraunhofer diffraction pattern of an arbitrary input signal with adjustable magnification. Further analysis of the system permits us to obtain a simple analytical expression to evaluate both the longitudinal and the transversal residual chromatic aberration, resulting in a compact achromatic Fourier transformer with low chromatic errors, even for a wide spectral content of the point source.

It is well known that the recording of a diffraction pattern of an input object illuminated by a polychromatic point source is chromatically blurred as a result of the wavelength dependence of the diffraction phenomenon. Achromatic processors are designed to compensate for the chromatic dispersion produced by the broadband illumination, permitting simultaneous achromatization of the entire diffraction pattern. In this way the optical Fourier transform provided by an ideal wavelength-independent Fourier transformer is located in a single Fraunhofer plane and has the same lateral magnification for all the spectral components of the point source.[1]

Diffractive optical elements have some potential advantages over conventional refractive or reflective components.[2] On the other hand, the use of diffractive elements seems to be limited to quasi-monochromatic optical systems because of the severe chromatic aberrations of these elements. However, some appropriate combinations of planar holographic lenses and refractive achromatic objectives permitted researchers to obtain achromatic imaging setups producing real images.[3,4] Optical achromatic Fourier transformers that contain holographic elements were also reported.[5–9] In the latter case the minimum number of optical components is three, and at least two of them are diffractive zone lenses.

In this Letter we present an all-diffractive achromatic Fourier-transform system based on white-light converging spherical-wave illumination. Our optical device simply consists of two on-axis blazed zone plates (ZP's). Here we again perform wavelength compensation, taking advantage of the chromatic aberrations of diffractive optical elements. The above system provides the achromatic Fourier transform (AFT) of the input signal at a finite distance, with low chromatic errors over the entire visible spectrum.

An analysis of the system leads to a simple relationship for evaluating in terms of a geometrical description both the longitudinal and the transversal residual chromatic aberration. Moreover, our proposed device uses two commercial diffractive elements and does not require any dispersive or achromatic glass objective, and thus it can be used in other ranges of the electromagnetic spectrum, say, in soft x rays. Finally, our proposal also has the following remarkable feature: we can vary the scale factor of the Fourier transform by simply moving the input signal along the optical axis of the system, with the achromatism remaining unchanged.

Let us remember that a blazed ZP has an associated image focal length $Z = Z_0\sigma/\sigma_0$ that is proportional to the wave number σ of the incident light. The constant Z_0 is simply the value of the focal length for the reference wave number σ_0.

To discuss the key for implementing our achromatic Fourier transformer we first recognize, using elementary geometrical-optics concepts, that only a set of chromatic planar objects forming a frustum of a right cone, as is indicated in Fig. 1, can be imaged by a single ZP in an achromatic image. The achromatic picture is achieved at an arbitrary distance d_1' if (i) the optical center of the ZP coincides with the axial point from which all the objects subtend the same angle, and (ii) the distances to the ZP from the objects are given by

$$d_1(\sigma) = \frac{Z_0 d_1' \sigma}{\sigma_0 d_1' - Z_0\sigma}. \tag{1}$$

Second, we reformulate the Fourier-transforming property of a ZP as follows: Let an input transparency be illuminated by a polychromatic spherical wave-front beam having a spectral bandwidth $\sigma_2 - \sigma_1$, as shown in Fig. 2. By use of the Fresnel diffraction theory, it is a straightforward matter to show that, for each spectral component of the incoming light, the ZP provides the Fraunhofer diffraction pattern of the input signal at the corresponding conjugate plane of the source plane, i.e., at a distance d' from the ZP given by

$$d'(\sigma) = \frac{Z_0 d\sigma}{\sigma_0 d - Z_0\sigma}, \tag{2}$$

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or, equivalently, at a distance $d + d'$ from point source S such that

$$d + d'(\sigma) = \frac{d^2\sigma_0}{\sigma_0 d - Z_0\sigma}. \qquad (3)$$

The notation is illustrated in Fig. 2. The scale factor of the Fourier transformation, which is evaluated for each wave number at the corresponding Fraunhofer plane, is

$$\frac{x}{u} = \frac{y}{v} = \frac{z}{\sigma}\frac{d'(\sigma)}{d} = \frac{Z_0 z}{\sigma_0 d - Z_0\sigma}, \qquad (4)$$

where x and y are Cartesian coordinates and u and v are spatial frequencies. It is important to recognize that the ratio $x/[d + d'(\sigma)]u$ or $y/[d + d'(\sigma)]v$ is independent of σ. From Eqs. (3) and (4) we conclude that

$$\frac{x}{[d + d'(\sigma)]u} = \frac{y}{[d + d'(\sigma)]v} = \frac{Z_0 z}{d^2\sigma_0}. \qquad (5)$$

Therefore the set of monochromatic versions of the Fourier transform of the input forms a frustum of a right cone whose apex coincides with point source S (see Fig. 2).

Consequently just a second blazed ZP with focal distance Z_0' for $\sigma = \sigma_0$, inserted at the source plane (a fact that implies converging spherical-wave illumination, i.e., $d < 0$), fulfills condition (i) and thus is able to recombine such monochromatic images into a single picture, providing the achromatic representation of the Fourier transform. The outline of our suggested optical configuration is depicted in Fig. 3.

Condition (ii) also must be fulfilled for the AFT to be achieved. If we want to obtain an AFT at a distance D_0' from the second zone plate, ZP_2, then Eq. (1) should be rewritten as

$$d_1(\sigma) = \frac{Z_0' D_0' \sigma}{\sigma_0 D_0' - Z_0'\sigma}. \qquad (6)$$

Thus in order to obtain our goal we set

$$d + d'(\sigma) = -d_1(\sigma). \qquad (7)$$

Nevertheless, comparing Eqs. (3) and (6) we see that Eq. (7) has no solution because the functional dependences of $d + d'$ and d_1 on σ are different. Alternatively we develop a first-order theory. We replace Eq. (7) by the two less restrictive conditions

$$d + d'(\sigma_0) = -d_1(\sigma_0), \qquad \dot{d}'(\sigma_0) = -\dot{d}_1(\sigma_0), \qquad (8)$$

where $\dot{x} = dx/d\sigma$. The solution of this equation system leads to the constraint that

$$Z_0' = \frac{-d^2}{Z_0}, \qquad (9)$$

which links the focal length of both ZP's with the separation between them, and the AFT is obtained at a distance D_0' such that

$$D_0' = \frac{d^2}{d - 2Z_0}. \qquad (10)$$

To obtain a real AFT, i.e., $D_0' > 0$, from Eqs. (9) and (10) we infer that ZP_1 and ZP_2 should be a diverging and a converging ZP, respectively, and that the value of the dimensionless parameter α, defined as

$$\alpha = \left|\frac{Z_0'}{Z_0}\right|, \qquad (11)$$

must be such that $0 < \alpha < 4$.

Since we have developed a first-order theory the proposed setup suffers from residual chromatic aberrations. In order to evaluate them we consider that ZP_2 images the diffraction volume generated by ZP_1. Using the Gaussian lens formula, we have

$$\frac{-1}{d + d'(\sigma)} + \frac{1}{D'} = \frac{\sigma_0}{Z_0'\sigma}, \qquad (12)$$

where, of course, D' is the distance from ZP_2 at which the final Fraunhofer plane is located for each σ. Substituting Eqs. (3) and (9) into Eq. (12) and operating, we obtain

$$D' = \frac{D_0'}{1 + \dfrac{1}{2 - \sqrt{\alpha}}\dfrac{(\sigma - \sigma_0)^2}{\sigma\sigma_0}}. \qquad (13)$$

In the above calculations we have used the equality

$$\frac{Z_0 D_0'}{d^2} = \frac{1}{\sqrt{\alpha} - 2}, \qquad (14)$$

which we easily derived, taking into account Eqs. (9)–(11). As we expect, the position of the

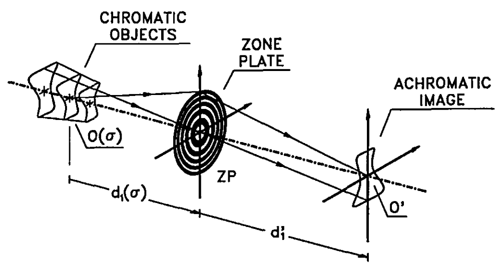

Fig. 1. Achromatic image produced by a blazed ZP.

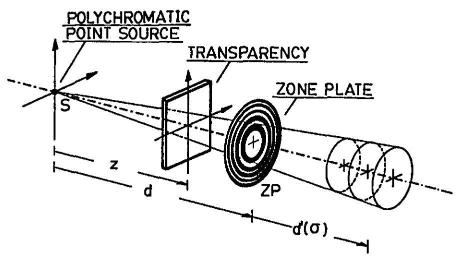

Fig. 2. Fourier-transforming properties of a blazed ZP under polychromatic spherical-wave illumination.

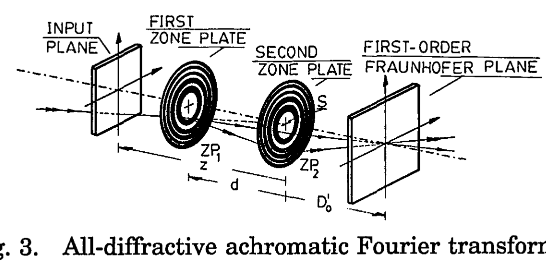

Fig. 3. All-diffractive achromatic Fourier transformer.

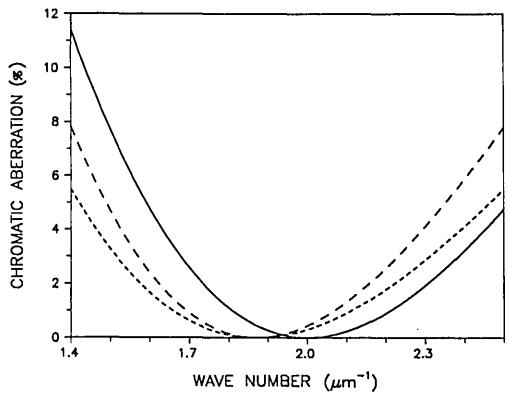

Fig. 4. Plot of the geometrical residual CA of the optical Fourier transformer in Fig. 3. Solid curve, $\alpha = 1$ and $\sigma_0 = 2\ \mu m^{-1}$; long-dashed curve, $\alpha = 1$ and $\sigma_0 = 1.87\ \mu m^{-1}$; short-dashed curve, $\alpha = 0.3$ and $\sigma_0 = 1.87\ \mu m^{-1}$.

final Fraunhofer diffraction pattern depends on σ. This is equivalent to saying that the plane in which the rigorous Fourier transform appears is slightly different for each σ. Of course, for $\sigma = \sigma_0$, D' becomes D_0', and $D' < D_0'$ for the rest of the wave numbers. Hence the final image volume is folded about the plane located at the distance D_0'. Consequently the effective diffraction pattern at the above plane is not an ideal wavelength-independent Fourier transform but an achromatic version of the Fraunhofer diffraction pattern of the transparency.

The scaling of the Fourier transform is $x' = Mx$, $y' = My$, where x and y are given by Eq. (4) and the lateral magnification M is such that $M = D'(\sigma)/[d + d'(\sigma)]$. Taking into account Eqs. (5), (13), and (14), we see that

$$\frac{x'}{u} = \frac{y'}{v} = \frac{-z/\sigma_0}{(2 - \sqrt{\alpha}) + \dfrac{(\sigma - \sigma_0)^2}{\sigma\sigma_0}}. \qquad (15)$$

Note that, in general, a quadratic phase error, which is different for each wave number, multiplies the different monochromatic replicas of the Fourier transform. In the present study we do not take into account the above phase curvature since we are interested in an AFT in intensity.

A good indication of the longitudinal chromatic aberration (LCA), expressed as a percentage, could be given by the fractional difference

$$\mathrm{LCA} = 100\frac{D_0' - D'(\sigma)}{D_0'}. \qquad (16)$$

Similarly we could define the transversal chromatic aberration (TCA) as

$$\mathrm{TCA} = 100\frac{x'(\sigma_0) - x'(\sigma)}{x'(\sigma_0)} = 100\frac{y'(\sigma_0) - y'(\sigma)}{y'(\sigma_0)}. \qquad (17)$$

From Eqs. (13) and (15) it is straightforward to show that both geometrical chromatic errors have an identical analytical expression. We will refer to either of them as the geometrical chromatic aberration (CA) of the setup and now have

$$\mathrm{CA} = \frac{100}{1 + (2 - \sqrt{\alpha})\dfrac{\sigma\sigma_0}{(\sigma - \sigma_0)^2}}. \qquad (18)$$

The variation of the residual chromatic aberration versus σ is dependent only on the value of α and the choice of the parameter σ_0. The function CA versus σ for three different pairs of values of the parameters α and σ_0 is plotted in Fig. 4. In this plot we assume that the spectral content of the incident light is the entire visible region, i.e., $\sigma_1 = 1.4\ \mu m^{-1}$ and $\sigma_2 = 2.5\ \mu m^{-1}$. It appears that having α less than 1 and a proper selection of the value of σ_0 is enough to achieve a chromatic error less than 8%, even with white light.

Concerning the scale factor, inspection of Eqs. (15) and (18) reveals that the scale factor is proportional to the distance z, and therefore it is a linear function of the longitudinal position of the input, but the chromatic aberration is independent of it. Thus for the first time to our knowledge an achromatic scale-tunable Fourier transformer is obtained.

The achromatic Fourier transformer we propose permits the extension of some of the conventional monochromatic information-processing techniques to polychromatic signal processing, and consequently full-color signals could be employed as input objects. In other words, the above setup can be thought of as a first stage in the design of achromatic white-light optical processors.

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